

# MIN-MAX EDGE ANTIMAGIC LABELING ON BOOK GRAPH

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## ABSTRACT

In this article the existences of odd min-max edge, even min-max edge, in-max edge Antimagic labeling of Book graph is shown.

**Key:** Magic, Labeling, min-max edge labeling, Odd, Even etc.

## I. INTRODUCTION

A labeling of a simple graph  $G$  is an assignment of integers to the vertices or edges or both subject to certain conditions. Most graph labeling methods extract their origin from a paper published by ROSA in 1967. Rosa called a function of  $\beta$ -valuation of a graph  $G$  with  $q$  edges of  $f$  is an injection from the vertices of  $G$  to the set  $\{0, 1, 2, 3, \dots, q\}$  such that when each edge  $uv$  is assigned the label  $|f(u) - f(v)|$  the resulting edge labels are distinct. Dynamic survey on Labeling is an important survey to know about the different labeling.[3]  
 Min-Max labeling was introduced by J.Jayapriya in the year 2016 [4]. In this paper the existences of min-max edge Antimagic labeling of Book graph is shown.

## II. BASICS DEFINITION

**Definition 2.1.** Let  $G(V, E)$  be a simple graph with  $p$  vertices and  $q$  edges.

A bijection  $f: V(G) \rightarrow \{1, 2, \dots, p\}$ , is said to be min-max edge antimagic labeling if for every edge  $uv$  in  $E$ , the real valued weight  $\lambda(uv) = \frac{\min\{f(u), f(v)\}}{\max\{f(u), f(v)\}}$  are distinct.[4]

**Definition 2.2.** Let  $G(V, E)$  be a simple graph with  $p$  vertices and  $q$  edges. A bijective function  $f: V(G) \rightarrow \{1, 3, 5, \dots, 2p-1\}$  is said to be odd min-max edge antimagic labeling if for every edge  $uv$  in  $E$ , the edge weights

$$\lambda(uv) = \frac{\min\{f(u), f(v)\}}{\max\{f(u), f(v)\}} \text{ are distinct. [4]}$$

**Definition 2.3.** A bijective function  $f: V(G) \rightarrow \{2, 4, 6, \dots, 2p\}$  is said to be even min-max edge antimagic

labeling if for every edge  $uv$  in  $E$ , the edge weights  $\lambda(uv) = \frac{\min\{f(u), f(v)\}}{\max\{f(u), f(v)\}}$  are distinct.[4]

**Definition 2.4.** Book graph is defined as the graph Cartesian product  $S_{m+1} \square P_2$ .

### III.MAIN RESULT

#### Theorem 1:

The  $B_6$  graph admits min-max edge, odd min-max, even min-max edge anti-magic labeling.

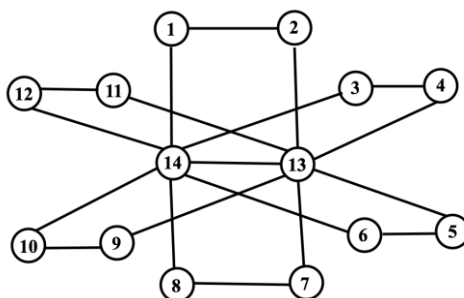
#### Proof:

Let  $G \{V, E\}$  be the  $B_6$  graph with  $V = \{v_1, v_2, \dots, v_{14}\}$  and  $E = \{e_1, e_2, \dots, e_{19}\}$

#### Case (1):

Let  $f: V \rightarrow \{1, 2, \dots, 14\}$  such that  $f(v_i) = i, 1 \leq i \leq 14$ .

Also, the edges are connected with those vertices such that the adjacent vertex labeling say  $f(v_i), f(v_j)$  for  $1 \leq i, j \leq 14$  are co-prime.



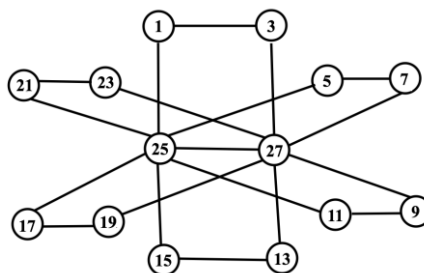
**Figure 3.1: Antimagic labeling of  $B_6$  graph**

In the above figure, the edge vertices are

$\{0.5, 0.07, 0.15, 0.75, 0.21, 0.31, 0.83, 0.38, 0.43, 0.875, 0.54, 0.57, 0.9, 0.69, 0.71, 0.92, 0.79, 0.85, 0.86\}$

#### Case (2):

Let  $f: V \rightarrow \{1, 3, 5, \dots, 31\}$ ,  $1 \leq i \leq 14$  such that  $f(v_i) = 2i-1, 1 \leq i \leq 14$ .



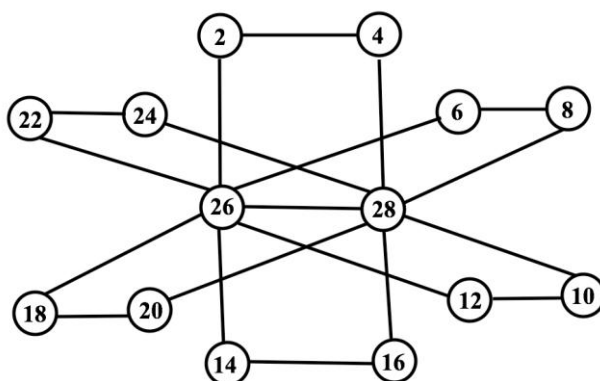
**Figure 3.2 : Odd Antimagic labeling of  $B_6$  graph**

The edge values are

$\{0.33, 0.11, 0.04, 0.71, 0.2, 0.26, 0.82, 0.33, 0.44, 0.87, 0.48, 0.6, 0.89, 0.70, 0.68, 0.91, 0.84, 0.85, 0.93\}$ .

**Case (3):**

Let  $f: V \rightarrow \{2, 4, 6, \dots, 28\}$ ,  $1 \leq i \leq 14$  such that  $f(v_i) = 2i$ ,  $1 \leq i \leq 14$ .



**Figure 3.3 : Even Antimagic labeling of  $B_6$  graph**

The edge values are

$\{0.5, 0.14, 0.07, 0.75, 0.23, 0.29, 0.83, 0.38, 0.46, 0.875, 0.57, 0.54, 0.9, 0.69, 0.71, 0.92, 0.85, 0.86, 0.93\}$ .

Then it is observed that the edge values are distinct.

Hence the min-max edge Antimagic labeling exists.

**Theorem 2:**

The  $B_5$  graph admits min-max edge, odd min-max, even min-max edge anti-magic labeling.

**Proof:**

Let  $G = \{V, E\}$  be the  $B_5$  graph with  $V = \{v_1, v_2, \dots, v_{12}\}$  and  $E = \{e_1, e_2, \dots, e_{16}\}$

**Case (1):**

Let  $f: V \rightarrow \{1, 2, \dots, 12\}$  such that  $f(v_i) = i$ ,  $1 \leq i \leq 12$ .

Also, the edges are connected with those vertices such that the adjacent vertex labeling say  $f(v_i)$ ,  $f(v_j)$  for  $1 \leq i, j \leq 12$  are co-prime.

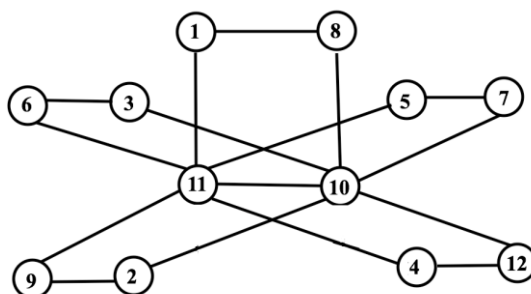


Figure 3.4 : Antimagic labeling of  $B_5$  graph

In the above figure, the edge vertices are

$\{0.125, 0.8, 0.91, 0.09, 0.3, 0.55, 0.5, 0.71, 0.45, 0.7, 0.83, 0.33, 0.36, 0.2, 0.22, 0.82\}$

Case (2):

Let  $f: V \rightarrow \{1, 3, 5, \dots, 23\}$ ,  $1 \leq i \leq 12$  such that  $f(v_i) = 2i-1$ ,  $1 \leq i \leq 12$ .

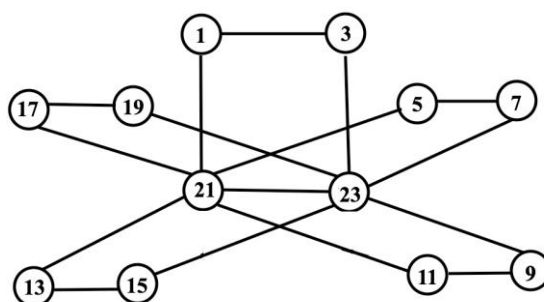


Figure 3.5 : Antimagic labeling of  $B_5$  graph

The edge values are  $\{0.33, 0.13, 0.91, 0.05, 0.83, 0.81, 0.89, 0.24, 0.71, 0.30, 0.39, 0.81, 0.52, 0.65, 0.87, 0.62\}$ .

Case (3):

Let  $f: V \rightarrow \{2, 4, 6, \dots, 24\}$ ,  $1 \leq i \leq 12$  such that  $f(v_i) = 2i$ ,  $1 \leq i \leq 12$ .

The edge values are  $\{0.6, 0.5, 0.92, 0.91, 0.18, 0.29, 0.58, 0.25, 0.375, 0.73, 0.33, 0.44, 0.82, 0.45, 0.2, 0.08\}$ .

Then it is observed that the edge values are distinct.

Hence the min-max edge Antimagic labeling.

**Theorem 3:**

The  $B_4$  graph admits min-max edge, odd min-max, even min-max edge anti-magic labeling.

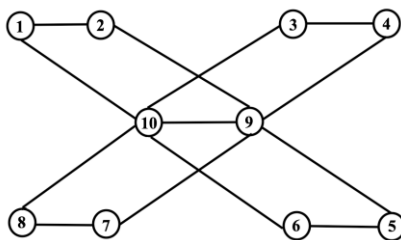
**Proof:**

Let  $G \{V, E\}$  be the  $B_4$  graph with  $V = \{v_1, v_2, \dots, v_{10}\}$  and  $E = \{e_1, e_2, \dots, e_{13}\}$

**Case (1):**

Let  $f: V \rightarrow \{1, 2, \dots, 10\}$  such that  $f(v_i) = i, 1 \leq i \leq 10$ .

Also, the edges are connected with those vertices such that the adjacent vertex labeling say  $f(v_i), f(v_j)$  for  $1 \leq i, j \leq 10$  are co-prime.



**Figure 3.6: Antimagic labeling of  $B_4$  graph**

In the above figure, the edge vertices are  $\{0.5, 0.1, 0.22, 0.75, 0.3, 0.44, 0.83, 0.55, 0.6, 0.875, 0.78, 0.8, 0.9\}$

**Case (2):**

Let  $f: V \rightarrow \{1, 3, 5, \dots, 19\}$ ,  $1 \leq i \leq 10$  such that  $f(v_i) = 2i-1, 1 \leq i \leq 10$ .

The edge values are  $\{0.33, 0.18, 0.05, 0.71, 0.26, 0.41, 0.65, 0.47, 0.82, 0.87, 0.88, 0.68, 0.81\}$ .

**Case (3):**

Let  $f: V \rightarrow \{2, 4, 6, \dots, 20\}$ ,  $1 \leq i \leq 10$  such that  $f(v_i) = 2i, 1 \leq i \leq 10$ .

The edge values are  $\{0.5, 0.22, 0.1, 0.75, 0.3, 0.44, 0.83, 0.55, 0.6, 0.875, 0.78, 0.8, 0.9\}$ .

Then it is observed that the edge values are distinct.

Hence the min-max edge Antimagic labeling exists.

## **IV.CONCLUSION**

The existences of Odd min-max, even min-max and min-max edge Antimagic labeling is shown.

## **REFERENCES**

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