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Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Spaces Using Implicit Relations

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ABSTRACT

In this paper, we give some concept of compatible and weak compatible mappings and prove a fixed point theorem in intuitionistic fuzzy metric spaces under the condition of weak compatible mapping by using implicit relations.

Keywords: coincidence point, common fixed point, intuitionistic fuzzy metric space, weak compatible maps.

2000 Mathematics Subject Classification: 47H10, 54H25.

I.INTRODUCTION

In 1965 the notion of fuzzy sets was initially investigated by Zadeh [15]. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. As a generalization of fuzzy sets, Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [14]. In 2004, Park [10] defined the notion of intuitionistic fuzzy metric space with the help of continuous *t*-norms and continuous *t*-conorms. Recently, in 2006, Alaca et al [3] defined with the help of continuous *t*-norm and continuous *t*-conorms as a generalization of fuzzy metric space, the idea of intuitionistic fuzzy sets and intuitionistic fuzzy metric space. Samanta and Mondal [11, 12] introduced the definition of the intuitionistic gradation of openness. In 2004, Park [10] introduced and discussed a notion of intuitionistic fuzzy metric spaces (briefly, IFM-spaces), which is based both on the idea of intuitionistic fuzzy sets and the concept of a fuzzy metric space given by George and Veeramani [6]. Kramosil & Michlek [9] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach [5], Turkoglu et al [13] gave the generalization of Jungck's [7] common fixed point theorem to intuitionistic fuzzy metric spaces, they first formulate the definition of weakly commuting and R-weakly commuting mapping in intuitionistic fuzzy metric space. The concept of compatible maps and compatible maps of type (A) and (B) was first formulated by Turkoglu, at. al [14] in intuitionistic fuzzy metric space. The aimed of this paper, we gave some concept of compatible and weak compatible mapping and we prove a fixed point theorem in intuitionistic fuzzy metric spaces under the condition of weak compatible mappings using implicit relations.

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II. PRELIMINARIES

DEFINITION (2.1)[10]: A binary operation *: [0, 1] × [0, 1]→[0, 1] is continuous t-norm if * is satisfying the following conditions:

(i) * is commutative and associative;

(ii) * is continuous;

(iii) a * 1 = a for all a∈ [0, 1];

(iv) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d∈ [0, 1].

DEFINITION (2.2)[10]: A binary operation ◊: [0, 1] × [0, 1]→[0, 1] is continuous t-conorm if ◊ is satisfying the following conditions:

(i) ◊ is commutative and associative;

(ii) ◊ is continuous;

(iii) a ◊ 0 = a for all a ∈ [0, 1];

(iv) a ◊ b ≥ c ◊ d whenever a ≤ c and b ≤ d for all a, b, c, d∈ [0, 1].

DEFINITION (2.3)[4]: A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions:

(i) M(x, y, t) + N(x, y, t) ≤ 1 for all x, y ∈ X and t > 0;

(ii) M(x, y, 0) = 0 for all x, y ∈ X;

(iii) M(x, y, t) = 1 for all x, y ∈ X and t > 0 if and only if x = y;

(iv) M(x, y, t) = M(y, x, t) for all x, y ∈ X and t > 0;

(v) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s) for all x, y, z ∈ X and s, t > 0;

(vi) For all x, y ∈ X, M(x, y, ·): [0, ∞)→[0, 1] is continuous;

(vii) $\lim_{t\to\infty} M$(x, y, t) = 1 for all x, y∈ X and t > 0;

(viii) N(x, y, 0) = 1 for all x, y ∈ X;

(ix) N(x, y, t) = 0 for all x, y∈ X and t > 0 if and only if x = y;

(x) N(x, y, t) = N(y, x, t) for all x, y∈ X and t > 0;

(xi) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s) for all x, y, z∈ X and s, t > 0;

(xii) For all x, y ∈ X, N(x, y, ·) : [0, ∞)→[0, 1] is continuous;

(xiii) $\lim_{t\to\infty} N$(x, y, t) = 0 for all x, y in X;

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

REMARK (2.1): Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *, ◊) such that t-norm * and t-conorm ◊ are associated as x ◊ y = 1- ((1-x) * (1-y)) for all x, y∈ X.

REMARK (2.2): In intuitionistic fuzzy metric space X, M(x, y, ·) is non-decreasing and N(x, y, ·) is non-increasing for all x, y ∈ X.

EXAMPLE (2.1): Let (x, d) be a metric space, define t-norm a * b = min {a, b} and t-conorm a ◊ b = max {a, b} and for all x, y ∈ X and t > 0,

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then (X, M, N, *, ◊) is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric.

DEFINITION (2.4)[4]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,

$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point x∈ X if, for all t > 0,

$\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$.

Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi) of definition (3), respectively.

DEFINITION (2.5)[4]: An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

DEFINITION (2.6)[14]: Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself. Then the maps A and B are said to be compatible if, for all t > 0,

$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ and $\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some x∈ X.

DEFINITION (2.7)[8]: Two self maps A and B in a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be weak compatible if they commute at their coincidence points. i.e. Ax = Bx for some x in X, then ABx = BAx.

DEFINITION (2.8)[6]: Let (X, M, N, *,◊) be an intuitionistic fuzzy metric space. A and B be self maps in X. Then a point x in X is called a coincidence point of A and B iff Ax = Bx. In this case y = Ax = Bx is called a point of coincidence of A and B.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

DEFINITION (2.9)[1]: Two self maps A and B in a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be occasionally weakly compatible (owc) iff there is a point x in X which is coincidence point of A and B at which A and B commute.

LEMMA (2.1)[4]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. if there exists a number k∈ (0, 1), such that $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$ and $N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$ for all t > 0 and n = 1, 2, … , then $\{y_n\}$ is a Cauchy sequence in X.

LEMMA (2.2)[13]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and for all x, y in X, t > 0 and if there exists a number k∈ (0, 1), M(x, y, kt) ≥ M(x, y, t) and N(x, y, kt) ≤ N(x, y, t), then x = y.

III.MAIN RESULT

IMPLICIT RELATIONS: Let $\boldsymbol{\phi}$ be the set of all continuous and increasing functions ϕ: [0, 1] $\rightarrow$ [0, 1], in each coordinate and $\phi(t) > t$ for all $t \in [0, 1)$. And also let $\boldsymbol{\psi}$ be the set of all continuous and decreasing functions $\psi : [0, 1] \rightarrow [0, 1]$ in each coordinates and $\phi(t) < t$, for all $t \in [0, 1)$.

THEOREM (3.1): Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space. Let A, B, S and T be mappings from X into itself satisfying:

(3.1) $S(X) \subset B(X)$ and $T(X) \subset A(X)$;

(3.2) if one of A(X), B(X), S(X) and T(X) is complete subset of X;

(3.3) A and S have a coincidence point;

(3.4) B and T have a coincidence point;

(3.5) there exists $k \in (0, \frac{1}{2})$ and $t > 0$ such that

$M(Sx, Ty, kt) \geq \phi(\text{Min}\{M(Ax, By, t), M(Ax, Sx, t), M(By, Ty, t), M(By, Sx, \alpha t), M(Ax, Sy, (2-\alpha)t)\})$

and $N(Sx, Ty, kt) \leq \psi(\text{Max}\{N(Ax, By, t), N(Ax, Sx, t), N(By, Ty, t), N(By, Sx, \alpha t), N(Ax, Sy, (2-\alpha)t)\})$

for all $x, y \in X$, $\alpha \in (0, 2)$ and $\phi \in \boldsymbol{\phi}$, $\psi \in \boldsymbol{\psi}$. If the pair (A, S) and (B, T) are weakly compatible then A, B, S and T have a unique common fixed point in X.

PROOF: Since we have $S(X) \subset B(X)$ and $T(X) \subset A(X)$, so we define two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_{2n+1} = Sx_{2n} = Bx_{2n+1}$, $y_{2n+2} = Tx_{2n+1} = Ax_{2n+2}$ (3.6)

Now, we take $x = x_{2n}$ and $y = x_{2n+1}$ in (3.5), we get

$M(Sx_{2n}, Tx_{2n+1}, kt) \geq \phi(\text{Min}\{M(Ax_{2n}, Bx_{2n+1}, t), M(Ax_{2n}, Sx_{2n}, t), M(Bx_{2n+1}, Tx_{2n+1}, t), M(Bx_{2n+1}, Sx_{2n}, \alpha t),$
$M(Ax_{2n}, Sx_{2n+1}, (2-\alpha)t)\})$

and $N(Sx_{2n}, Tx_{2n+1}, kt) \leq \psi(\text{Max}\{N(Ax_{2n}, Bx_{2n+1}, t), N(Ax_{2n}, Sx_{2n}, t), N(Bx_{2n+1}, Tx_{2n+1}, t), N(Bx_{2n+1}, Sx_{2n}, \alpha t),$
$N(Ax_{2n}, Sx_{2n+1}, (2-\alpha)t)\})$.

For $\alpha = 1$ and by (3.6), we get

$M(y_{2n+1}, y_{2n+2}, kt) \geq \phi(\text{Min}\{M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n+1}, y_{2n+1}, \alpha t),$
$M(y_{2n}, y_{2n+2}, (2-\alpha)t)\})$
$\geq \phi(\text{Min}\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, 1, M(y_{2n}, y_{2n+2}, t)\})$
$\geq \phi(\text{Min}\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, M(y_{2n}, y_{2n+2}, t)\})$
$\geq \phi(\text{Min}\{M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t)\})$
$\geq \phi(M(y_{2n}, y_{2n+1}, t))$

and $N(y_{2n+1}, y_{2n+2}, kt) \leq \psi(\text{Max}\{N(y_{2n}, y_{2n+1}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), N(y_{2n+1}, y_{2n+1}, \alpha t),$
$N(y_{2n}, y_{2n+2}, (2-\alpha)t)\})$
$\leq \psi(\text{Max}\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, 0, N(y_{2n}, y_{2n+2}, t)\})$
$\leq \psi(\text{Max}\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, N(y_{2n}, y_{2n+2}, t)\})$
$\leq \psi(\text{Max}\{N(y_{2n}, y_{2n+1}, t), N(y_{2n}, y_{2n+1}, t)\})$
$\leq \psi(N(y_{2n}, y_{2n+1}, t))$.

In view of ϕ and ψ, we get

$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$ and $N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t)$.

By lemma (2.1), we get $\{y_n\}$ is Cauchy sequence in X and it converges to a point z in X.

Therefore, its subsequences $\{y_{2n}\}$, $\{y_{2n+1}\}$, $\{y_{2n+2}\}$ are also converges to a point z in X. That is,

$Ax_{2n+2} = Bx_{2n+1} = Sx_{2n} = Tx_{2n+1} = z$ as $n \rightarrow \infty$.

Now, we suppose that A(X) is complete subset of X. Then we get $Aw = z$. (3.7)

Now, we take $x = w$ and $y = x_{2n+1}$ in (3.5), we get

$$M(Sw, Tx_{2n+1}, kt) \geq \phi(Min\{M(Aw, Bx_{2n+1}, t), M(Aw, Sw, t), M(Bx_{2n+1}, Tx_{2n+1}, t), M(Bx_{2n+1}, Sw, \alpha t), M(Aw, Sx_{2n+1}, (2-\alpha)t)\})$$

and $N(Sw, Tx_{2n+1}, kt) \leq \psi(Max\{N(Aw, Bx_{2n+1}, t), N(Aw, Sw, t), N(Bx_{2n+1}, Tx_{2n+1}, t), N(Bx_{2n+1}, Sw, \alpha t), N(Aw, Sx_{2n+1}, (2-\alpha)t)\})$.

Taking $n \rightarrow \infty$, we get

$M(Sw, z, kt) \geq \phi(Min\{M(Aw, z, t), M(Aw, Sw, t), M(z, z, t), M(z, Sw, \alpha t), M(Aw, z, (2-\alpha)t)\})$

and $N(Sw, z, kt) \leq \psi(Max\{N(Aw, z, t), N(Aw, Sw, t), N(z, z, t), N(z, Sw, \alpha t), N(Aw, z, (2-\alpha)t)\})$.

For $\alpha = 1$ and by (3.7), we get

$$\begin{aligned} M(Sw, z, kt) &\geq \phi(Min\{M(z, z, t), M(z, Sw, t), M(z, Sw, t), 1, M(z, Sw, t), M(z, z, t)\}) \\ &\geq \phi(Min\{1, M(z, Sw, t), M(z, Sw, t), 1\}) \\ &\geq \phi(M(z, Sw, t)) \end{aligned}$$

and
$$\begin{aligned} N(Sw, z, kt) &\leq \psi(Max\{N(z, z, t), N(z, Sw, t), N(z, Sw, t), 0, N(z, Sw, t), N(z, z, t)\}) \\ &\leq \psi(Max\{0, N(z, Sw, t), N(z, Sw, t), 0\}) \\ &\leq \psi(N(z, Sw, t)). \end{aligned}$$

In view of ϕ and ψ, we get $M(z, Sw, kt) \geq M(z, Sw, t)$ and $N(z, Sw, kt) \leq N(z, Sw, t)$

By lemma (2.2), we get $z = Sw$. That is $Sw = z = Aw$.

Therefore, w is coincidence point of A and S.

Now, since $S(X) \subset B(X)$, Therefore, $z = Sv \in S(X) \subset B(X)$, this gives $z \in B(X)$. Now let $Bv = z$,

we take $x = x_{2n}$ and $y = v$ in (3.5), we get

$M(Sx_{2n}, Tv, kt) \geq \phi(Min\{M(Ax_{2n}, Bv, t), M(Ax_{2n}, Sx_{2n}, t), M(Bv, Tv, t), M(Bv, Sx_{2n}, \alpha t), M(Ax_{2n}, Sv, (2-\alpha)t)\})$

and $N(Sx_{2n}, Tv, kt) \leq \psi(Max\{N(Ax_{2n}, Bv, t), N(Ax_{2n}, Sx_{2n}, t), N(Bv, Tv, t), N(Bv, Sx_{2n}, \alpha t), N(Ax_{2n}, Sv, (2-\alpha)t)\})$.

By (3.6) and for $Bv = z$, we have

$M(y_{2n+1}, Tv, kt) \geq \phi(Min\{M(y_{2n}, z, t), M(y_{2n}, y_{2n+1}, t), M(z, Tv, t), M(z, y_{2n+1}, \alpha t), M(y_{2n}, z, (2-\alpha)t)\})$

and $N(y_{2n+1}, Tv, kt) \leq \psi(Max\{N(y_{2n}, z, t), N(y_{2n}, y_{2n+1}, t), N(z, Tv, t), N(z, y_{2n+1}, \alpha t), N(y_{2n}, z, (2-\alpha)t)\})$.

For $\alpha = 1$ and taking $n \rightarrow \infty$, we get

$$\begin{aligned} M(z, Tv, kt) &\geq \phi(Min\{M(z, z, t), M(z, z, t), M(z, Tv, t), M(z, z, t), M(z, z, t)\}) \\ &\geq \phi(Min\{1, 1, M(z, Tv, t), 1, 1\}) \\ &\geq \phi(M(z, Tv, t)) \end{aligned}$$

and $N(z, Tv, kt) \leq \psi(Max\{N(z, z, t), N(z, z, t), N(z, Tv, t), N(z, z, t), N(z, z, t)\})$

$\leq \psi(Max\{0, 0, N(z, Tv, t), 0, 0\})$

$\leq \psi(N(z, Tv, t))$.

In view of ϕ and ψ, we get $M(z, Tv, kt) \geq M(z, Tv, t)$ and $N(z, Tv, kt) \leq N(z, Tv, t)$.

By lemma (2.2), we get z = Tv. That is Tv = z = Bv.

Therefore, v is coincidence point of T and B.

Now, since (A, S) is weakly compatible, therefore A and S commute at coincidence point. That is ASw = SAw, this gives Az = Sz. (3.8)

And (B, T) is weakly compatible, therefore BTv = TBv, this gives Bz = Tz. (3.9)

Now, firstly we will show thai Sz = z. Take x = z and y = x_{2n+1} in (3.5), we get

$M(Sz, Tx_{2n+1}, kt) \geq \phi(Min\{M(Az, Bx_{2n+1}, t), M(Az, Sz, t), M(Bx_{2n+1}, Tx_{2n+1}, t), M(Bx_{2n+1}, Sz, \alpha t),$
$M(Az, Sx_{2n+1}, (2\text{-}\alpha)t)\})$

and $N(Sz, Tx_{2n+1}, kt) \leq \psi(Max\{N(Az, Bx_{2n+1}, t), N(Az, Sz, t), N(Bx_{2n+1}, Tx_{2n+}1, t), N(Bx_{2n+1}, Sz, \alpha t),$
$N(Az, Sx_{2n+1}, (2\text{-}\alpha)t)\})$.

From (3.6) and (3.8), we get

$M(Sz, y_{2n+2}, kt) \geq \phi(Min\{M(Sz, y_{2n+1}, t), M(Sz, Sz, t), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n+1}, Sz, \alpha t), M(Sz, y_{2n+2}, (2\text{-}\alpha)t)\})$

and $N(Sz, y_{2n+2}, kt) \leq \psi(Max\{N(Sz, y_{2n+1}, t), N(Sz, Sz, t), N(y_{2n+1}, y_{2n+2}, t), N(y_{2n+1}, Sz, \alpha t),$
$N(Sz, y_{2n+2}, (2\text{-}\alpha)t)\})$.

For $\alpha = 1$ and taking $n \to \infty$, we get

$M(Sz, z, kt) \geq \phi(Min\{M(Sz, z, t), 1, M(z, z, t), M(z, Sz, t), M(Sz, z, t)\})$

$\geq \phi(M(Sz, z, t))$

and $N(Sz, z, kt) \leq \psi(Max\{N(Sz, z, t), 0, N(z, z, t), N(z, Sz, t), N(Sz, z, t)\})$

$\leq \psi(N(Sz, z, t))$.

In view of ϕ and ψ, we get $M(Sz, z, kt) \geq M(Sz, z, t)$ and $N(Sz, z, kt) \leq N(Sz, z, t)$.

By lemma (2.2), we get Sz = z. That is Sz = z = Az. (3.10)

Again we will show that Tz = z. Take x = x_{2n} and y = z in (3.5), we get

$M(Sx_{2n}, Tz, kt) \geq \phi(Min\{M(Ax_{2n}, Bz, t), M(Ax_{2n}, Sx_{2n}, t), M(Bz, Tz, t), M(Bz, Sx_{2n}, \alpha t), M(Ax_{2n}, Sz, (2\text{-}\alpha)t)\})$

and $N(Sx_{2n}, Tz, kt) \leq \psi(Max\{N(Ax_{2n}, Bz, t), N(Ax_{2n}, Sx_{2n}, t), N(Bz, Tz, t), N(Bz, Sx_{2n}, \alpha t),$
$N(Ax_{2n}, Sz, (2\text{-}\alpha)t)\})$.

From (3.6) and (3.9), we get

$M(y_{2n+1}, Tz, kt) \geq \phi(Min\{M(y_{2n}, Tz, t), M(y_{2n}, y_{2n+1}, t), M(Tz, Tz ,t), M(Tz, y_{2n+1}, \alpha t), M(y_{2n}, z, (2\text{-}\alpha)t)\})$

and $N(y_{2n+1}, Tz, kt) \leq \psi(Max\{N(y_{2n}, Tz, t), N(y_{2n}, y_{2n+1}, t), N(Tz, Tz ,t), N(Tz, y_{2n+1}, \alpha t), N(y_{2n}, z, (2\text{-}\alpha)t)\})$.

For $\alpha = 1$ and taking $n \to \infty$, we get

$M(z, Tz, kt) \geq \phi(Min\{M(z, Tz, t), M(z, z, t), 1, M(Tz, z, t), M(z, z, t)\})$

$\geq \phi(M(z, Tz, t))$

and $N(z, Tz, kt) \leq \psi(Max\{N(z, Tz, t), N(z, z, t), 0, N(Tz, z, t), N(z, z, t)\})$

$\leq \psi(N(z, Tz, t))$.

In view of ϕ and ψ, we get $M(z, Tz, kt) \geq M(z, Tz, t)$ and $N(z, Tz, kt) \leq N(z, Tz, t)$.

By lemma (2.2), we get $z = Tz$. That is $Tz = z = Bz$. (3.11)

Combining (3.10) and (3.11), we get $Az = Bz = Sz = Tz = z$.

Therefore, z is a common fixed point of A, B, S and T.

For uniqueness, let w be another fixed point of A, B, S and T. Then we have $Aw = Bw = Sw = Tw = w$.

Take $x = z$ and $y = w$ in (3.5), we get

$M(Sz, Tw, kt) \geq \phi(\text{Min}\{M(Az, Bw, t), M(Az, Sz, t), M(Bw, Tw, t), M(Bw, Sz, \alpha t), M(Az, Sw, (\alpha\text{-}2)t)\})$

$M(z, w, kt) \geq \phi(\text{Min}\{M(z, w, t), M(z, z, t), M(w, w, t), M(w, z, \alpha t), M(z, w, (\alpha\text{-}2)t)\})$

and $N(Sz, Tw, kt) \leq \psi(\text{Max}\{N(Az, Bw, t), N(Az, Sz, t), N(Bw, Tw, t), N(Bw, Sz, \alpha t), N(Az, Sw, (\alpha\text{-}2)t)\})$

$N(z, w, kt) \leq \psi(\text{Max}\{N(z, w, t), N(z, z, t), N(w, w, t), N(w, z, \alpha t), N(z, w, (\alpha\text{-}2)t)\})$.

For $\alpha = 1$, we get $M(z, w, kt) \geq \phi(M(z, w, t))$ and $N(z, w, kt) \leq \psi(N(z, w, t))$.

In view of ϕ and ψ, we get $M(z, w, kt) \geq M(z, w, t)$ and $N(z, w, kt) \leq N(z, w, t)$.

By lemma (2.2), we get $z = w$. Hence z is unique common fixed point of A, B, S and T.

If we take $T = S$ in Theorem (3.1), we have the following result.

COROLLARY (3.2): Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Let A, B and S be mappings from X into itself satisfying:

(3.12) $S(X) \subset B(X)$ and $S(X) \subset A(X)$;

(3.13) if one of A(X), B(X) and S(X) is complete subset of X;

(3.14) A and S have a coincidence point;

(3.15) B and S have a coincidence point;

(3.16) there exists $k \in (0, \frac{1}{2})$ and $t > 0$ such that

$M(Sx, Sy, kt) \geq \phi(\text{Min}\{M(Ax, By, t), M(Ax, Sx, t), M(By, Sy, t), M(By, Sx, \alpha t), M(Ax, Sy, (\alpha\text{-}2)t)\})$

and $N(Sx, Sy, kt) \leq \psi(\text{Max}\{N(Ax, By, t), N(Ax, Sx, t), N(By, Sy, t), N(By, Sx, \alpha t), N(Ax, Sy, (\alpha\text{-}2)t)\})$

for all $x, y \in X$, $\alpha \in (0, 2)$ and $\phi \in \boldsymbol{\phi}$, $\psi \in \boldsymbol{\psi}$. If the pair (A, S) and (B, S) are weakly compatible then A, B and S have a unique common fixed point in X.

EXAMPLE (3.1): Let $X = \{\frac{1}{2n}, n = 1, 2, 3, \ldots\} \cup \{0\}$ with the usual metric and, for all $t > 0$ and $x, y \in X$, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t>0 \\ 0, & t=0 \end{cases} \quad \text{and } N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t>0 \\ 1, & t=0. \end{cases}$$

Cleary, $(X, M, N, *, \Diamond)$ is a intuitionistic fuzzy metric space, where $*$ and $\Diamond$ are defined by $a * b = \text{Min}\{a, b\}$ and $a \Diamond b = \text{Max}\{a, b\}$ respectively. Let A, B, S and T be defined by

$Ax = \frac{2x}{8}$, $Sx = \frac{2x}{6}$ $Bx = \frac{2x}{12}$, $Tx = \frac{2x}{6}$ for all $x \in X$.

Then, we have $A(X) = \{\frac{2}{8n}, n = 1, 2, 3, \ldots\} \cup \{0\} \subseteq \{\frac{2}{4n}, n = 1, 2, 3, \ldots\} \cup \{0\} = S(X)$

B(X) = $\{\frac{2}{12n}, n = 1, 2, 3, \ldots\} \cup \{0\} \subseteq \{\frac{2}{6n}, n = 1, 2, 3, \ldots\} \cup \{0\}$ = T(X).

Also, the condition (3.5) of theorem (3.1) is satisfied and A, B, S and T are continuous, if ϕ is increasing in each of its coordinate and $\phi(t) > t$, and ψ is decreasing in each of its coordinates and $\psi(t) < t$ for all $t \in [0, 1)$. Further, the pairs (A, S) and (B, T) are weak compatible if

$\lim_{n\to\infty} x_n = 0$, where $\{x_n\}$ is a sequence in X, such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = 0$ for some $0 \in X$.

Thus all the conditions of Theorem (3.1) are satisfied and also 0 is the unique common fixed point of A, B, S and T.

REFERENCES

[1]. M. A. Al-Thaga and N. Shahzad: Generalised I-nonexpansive self maps and invariants approximations. Acta Math. Sin., 24(5):867:876, 2008.

[2]. C. Alaca: On fixed point theorems in intuitionistic fuzzy metric spaces, Commun. Korean Math. Soc. 24(2009), 565-579.

[3]. C. Alaca, D. Turkoglu, C. Yildiz: Common fixed points of compatible maps in intuitionistic fuzzy metric spaces, Southeast Asian Bull. Math. 32(2008), 21-33.

[4]. C. Alaca, D. Turkoglu and C. Yildiz: Fixed points in intuitionistic fuzzy metric spaces, Smallerit Choas, Solitons & Fractals, 29(5)(2006), 1073-1078.

[5]. S. Banach: Theories, lies, operations. Laniaries Manograie Mathematyezene, warsaw, Poland, 1932.

[6]. A. George, P. Veeramani: On some result in fuzzy metric space, Fuzzy Sets Systems 64(1994), 395-399.

[7]. G. Jungck: commuting mappings and fixed points Amer. Math. Monthly, 83(1976), 261- 263.

[8]. Jungck, G.: Compatible mappings and common fixed points, Internat. J. Math. Sci. 9(1986), 771-779.

[9]. J. Kramosil, J. Michalek: Fuzzy metric and Statistical metric spaces, Kybernetica, 11(1975), 326-334.

[10]. J. H. Park: Intuitionistic fuzzy metric spaces, Chaos Solit. Fract. 22(2004), 1039-1046.

[11]. S. K. Samanta, T. K. Mondal: Intuitionistic gradation of openness: intuitionistic fuzzy topology, Busefal 73(1997), 8-17.

[12]. S. K. Samanta, T. K. Mondal: On intuitionistic gradation of openness, Fuzzy Sets Syst.131(2002), 323-336.

[13]. D. Turkoglu, C. Alace and C. Yildiz: Compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces, Demonstratio Math. 39(2006), 671-684.

[14]. D.Turkoglu, I. Altun, and Y. J. Cho: Common fixed points of compatible mappings of type (I) and (II) in fuzzy metric spaces, J. Fuzzy Math. 15(2007), 435-448.

[15]. L. A. Zadeh: Fuzzy sets, Infor. and Control. 8 (1965), 338-353.