

USES OF GRAPH THEORY IN DAY TO DAY LIFE

Veena Rani

Department of Mathematics, Govt. National College, Sirsa (Haryana) (India)

ABSTRACT

Graph theory is a branch of discrete mathematics. Graphs provide a convenient way to represent various kinds of mathematical objects. There is wide use of graphs in providing problem solving techniques. This paper gives an overview of the uses of graph theory in heterogeneous fields such as chemistry, biology, computer science, mathematics etc. apart from brief discussion of problems in time table scheduling. Various papers based on graph theory have been studied related to its uses and scheduling concepts and an overview has been presented here besides discussion of its uses in our day to day life.

I. INTRODUCTION

While introducing Graph theory, it is worth to mention here that Graph theory started with the problem of Koinberg bridge, in 1735. The City of Koinberg had seven bridges on the Pregel River. The problem was that whether it would be possible to take a walk through the city passing exactly once on each bridge. This problem lead to the concept of Eulerian graph. Euler studied the problem and constructed a structure to solve the problem called Eulerian graph. He built the representative graph, observed that it had vertices of odd degree and proved that this made such a walk impossible. In 1840, A.F. Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problem. The concept of tree (a connected graph with cycles) was implemented by Gustav Kirchhoff in 1845, he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Guthrie found the famous four color problem and it was solved after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of graph theory.[1]

II. WHAT IS GRAPH

Definition 2.1: A graph is usually denoted by $G(V,E)$ consists of set of vertices V together with a set of edges E . Vertices are also known as nodes or points as actors, as agents or players (in social networks) and Edges are known as lines or ties or links. An edge $e(x,y)$ is defined by the unordered pair of vertices that serve as its end points.

Definition 2.2. Two vertices x and y are adjacent if there exist an edge (x,y) that connects them and edge is said to be incident upon nodes x and y .

Definition 2.3. A graph in which all vertices are adjacent to the remaining ones is said to be complete graph.

Definition 2.4. Number of vertices adjacent to a given vertex is called the degree of the vertex and is denoted by $d(x)$.

Definition 2.5. A bipartite graph, bigraph is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to a vertex in V .

Definition 2.6. An Eulerian circuit in a graph is a circuit which includes every vertex and every edge of G . It may pass through a vertex more than once but it crosses each edge exactly once. A graph which has a Eulerian circuit is called an Eulerian graph.

Definition 2.7. A Hamilton circuit is a path that visits every vertex in the graph exactly once and return to the starting vertex.[1]

III. USES OF GRAPH THEORY

Graph theoretical concepts are widely used to study and model various applications in different areas. It provides a convenient way to represent various kinds of mathematical objects. Graphs gives us many techniques and flexibility while defining and solving a real life problem. Mathematicians are concerned with the abstract structure of a graph. They analyse and manipulate graphs and develop theorems based upon structural axioms. Not only in Mathematics, graph theoretical concepts are widely used in various applications in different areas also. They include study of molecules, construction of bonds and study of atoms in chemistry. In biology, graph theory is used for conservation efforts where a vertex represent regions where certain species exist and the edges represent migration path or movement between them. This information is important when looking at breeding patterns or tracking the spread of diseases, parasites and to study the impact of migration that affect other species. Graph theory is used in sociology to measure actor's prestige or to explore diffusion mechanism. It is also used in computer applications for development of graph algorithms. Now we will discuss some problems and the related applications of graph theory to find their solutions.[2]

3.1 Graph Enumeration Technique: Graph enumeration technique is used to identify the computerized chemical identification. All structural formulas of covalently bounded compounds are graph; they are therefore, called molecular graphs or better constitutional graphs. From the chemical compounds described and indexed so far, more than 90% are organic or certain organic ligands in whose constitutional formulas the lines (the edges of graph) symbolize covalent two – electron bonds and the points (the vertices of graph) symbolize atoms or more exactly atomic course excluding the valence electrons. Graph theory provides the basis for definition, enumeration systemization, codification, nomenclature, co-relation and computer programming.

3.2 Graph Theory used in Computer Science: The major role of graph theory in computer applications is the development of graph algorithms. Numerous algorithms are used to solve problems that are modeled in the form of graphs. These algorithms are used to solve the graph theoretical concepts which in turn used to solve the corresponding computer science application problems. Some algorithms are as follows:

- 3.2.1. Shortest path algorithm in a network.
- 3.2.2. Finding graph planarity.
- 3.2.3. Finding a minimum spanning tree.
- 3.2.4. Algorithms to find connectedness.

- 3.2.5. Algorithms to find the cycles in a graph.
- 3.2.6. Algorithms to find adjacency matrices.
- 3.2.7. Algorithms for searching an element in a data structure and so on.

Various computer languages are used to support the graph theory concepts. The main goal of such languages is to enable the user to formulate operations on graphs in a compact and natural manner.[3]

3.3. Graph theory in OR: Graph theoretical concepts are widely used in Operational Research. It is used in modeling, transport networks, activity networks and theory of games. The network activity is used to solve large number of combinatorial problems. The most popular and successful applications of networks in OR is the planning and scheduling of large complicated projects. The best well known problems are PERT (Project Evaluation Review Technique) and CPM (Critical Path Method). Next, game theory is applied to the problems in engineering, economics and war science to find optimal way to perform certain tasks in competitive environments. To represent the method of finite game, a bigraph is used. Here, the vertices represent the positions and edges represent the moves. [4]

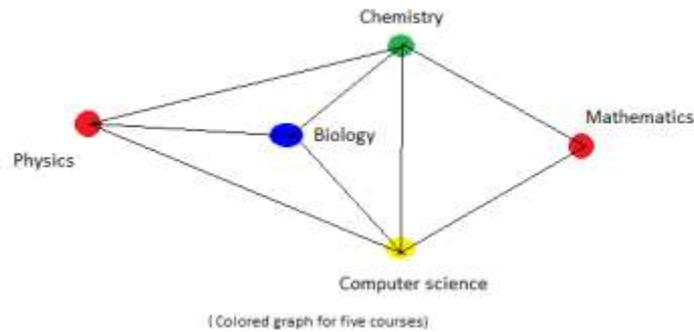
IV. GRAPH COLOURING TECHNIQUES:

Graph colouring is one of the most important concepts in graph theory. Proper coloring of a graph is the coloring of vertices and edges with minimal number of colors such that no two adjacent vertices should have the same color. This technique is used in many scheduling problems such as job scheduling, air craft scheduling, bi-processor tasks and timetabling etc. To understand it clearly, one of these scheduling problems, is explained here below:

4.1 Time table scheduling: Time table scheduling or timetabling is the most complex and error prone problem. Therefore, there is great requirement of an application which distribute the courses evenly and without collision. Graph coloring technique is one of the most used techniques for this. However, the particular form of time table required a specific to the environment or institutions in which it is needed. For example, a school timetabling problem and a university timetabling problem at the first instance seem to be the same problem but there are many differences. One such difference is that in a school class sizes are almost similar and all rooms are sufficiently large enough. On the other hand, in universities, class size can range from 05 to 200 students. Infact, problem may be changed even within one institution also. Graph coloring technique plays an important role in all such problems.

Problem 4.1.1:

A simple time table problem is one in which we have five courses to be scheduled: Physics, Chemistry, Biology, Mathematics and Computer Science. Given a graph showing the time table conflict between them where vertices represent subjects and edges between them represent conflict i.e. no two adjacent vertices can together be scheduled.



By Graph coloring technique we have assigned here colors as Red to Mathematics, Green to Chemistry, Yellow to Computer Science, Blue to Biology and again Red to Physics so that no two vertices of same colors are adjacent. The above colors represent periods in which the particular courses are to be scheduled. We get minimum number of colors is 04 i.e. four periods are required to meet all the subjects.

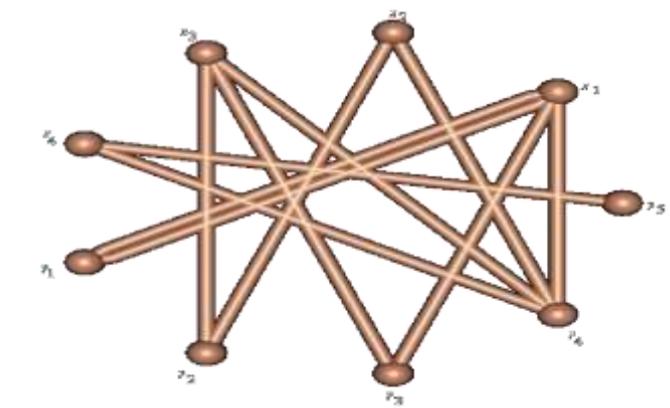
Problem 4.1.2:

In this problem timetabling of an institute is discussed in which there are four teachers x_1, x_2, x_3, x_4 and five subjects y_1, y_2, y_3, y_4, y_5 to be taught. The teacher requirement matrix is given as follows. To find the solution of time tabling problem, we use graph coloring technique.

(Teacher requirement matrix)

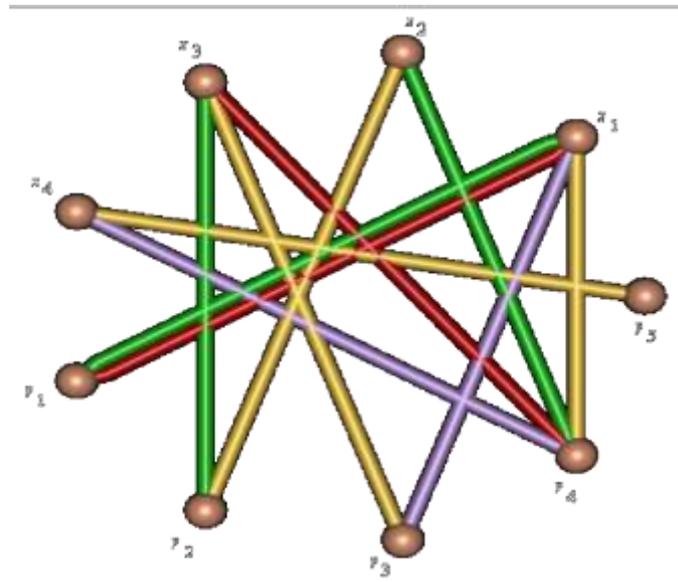
p	y_1	y_2	y_3	y_4	y_5
x_1	2	0	1	1	0
x_2	0	1	0	1	0
x_3	0	1	1	1	0
x_4	0	0	0	1	1

The bipartite graph is constructed with two sets U and V of teachers and subjects respectively.



(Figure 1. bipartite graph with four teachers and five subjects)

The proper coloring of the above mentioned graph can be done by four colors using the vertex coloring algorithm [5] which leads to the edge coloring of the bipartite multi graph as hereinbelow:



(Figure 2. Minimum proper coloring of edges of G)

Four colors in above proper coloring of edges of graph interpreted to four periods and finally the solution of time table is obtained.[6]

(The Time Table)

-	1	2	3	4
x_1	y_1	y_1	y_3	y_4
x_2	y_4	-	-	y_2
x_3	y_2	y_4	-	y_3
x_4	-	-	y_4	y_5

V. USES OF GRAPH THEORY IN DAY TO DAY LIFE

Perhaps we do not notice that in our day to day life, we are using graph theory. Infact graph theory is being used in our so many routine activities. For example:

5.1. Using GPS or Google maps/ Yahoo maps to determine a route based on user settings (quickest route/ shortest route) or finding the cheapest airfare between two destinations. The destinations are vertices and

their connections are edges containing information such as distance or air fare. The software finds the critical path (optimal route) based on the user settings.

- 5.2. Connecting with friends via social media or a video going viral. Each user is a vertex and when users connect, they create an edge. Videos are known to be viral when they have reached a certain number of connections/ views.
- 5.3. Using Google to search for web pages. Pages on the internet are linked to each other by hyperlinks; each page is a vertex and link between to pages is an edge. PageRank and Googlebot are used algorithms to aid the connectivity process.
- 5.4. School administration developing bus routes to pick up students to deliver the school. Each stop is a vertex and the route is an edge. A Hamiltonian path represents the efficiency of including every vertex in the route.
- 5.5. Visiting a zoo, water-park or theme park and wanting to see certain attractions or devise an efficient route to see all the attractions. A Hamiltonian path or circuit is used which contains every vertex in the graph.
- 5.6. City planning to put salt on the roads when ice develops. Euler paths or circuits are used to pass over the streets in the most efficient way.[7]

VI.CONCLUSION

In this paper brief introduction of graph theory and its applications has been discussed. It is beneficial for students of various streams such as Computer Science, Social Science, General Science etc. Time table scheduling problem has been widely discussed which is very helpful for any educational institution. Applications of graph theory are also useful in day to day life.

REFERENCES

- [1] *Introduction to Graph Theory* by Allen Dickson, October-2016.
- [2] *Graph Theory & Applications* by Paul Van Dooren, Universite Catholique de Louvai, Louvain-la-neuve, Belgium, August, 2009.
- [3] *International Journal of Computer Applications (0975-8887) Vol.104- No.(October 2014) (10-13).*
- [4] *International Journal of Engineering Science and Technology Vol.II(9),2010,4610-4621.*
- [5] Ashay Dharwadker, *The Vertex Coloring Algorithm, 2006.*
<http://www.dharwadker.org/vertexcoloringalgorithm>
- [6] *An Introduction to Timetabling* D.de WERRA Swiss Federal Institute of Technology, Department of Mathematics, CH-1015, Lavesanne, Switzerland.
- [7] *Graph Theory Problems and Solutions* by Tom Davis, November 11, 2006.