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Propagation of a Homogeneous Incompressible Impure Viscoelastic Fluid through a Circular Pipe under the Influence of Transverse Magnetic Field

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ABSTRACT

The aim of this paper is to analyze the flows of dusty viscoelastic fluids in circular pipe. We study this in steady state conditions under the influence of transverse magnetic field. Here the fluid is at the same temperature as the surroundings. Finite element method is used to solve the partial differential equation governing the flow of the fluid and the dust by converting them into ordinary differential equations. Analytical solutions for the velocity of fluid and dust particles are obtained. The results have been numerically evaluated and shown graphically and discussed.

1. INTRODUCTION

In recent years, the motion of a dusty viscoeleastic fluid through circular pipes has wide applications in the various fields of biomechanics and engineering. Some problems that involve the dynamic response of the fluid to the frequency of the periodic pressure gradient are normally occurring in chemical and petroleum industries. Fluids such as honey, oil, blood and some polymer solutions exhibit both viscous and elastic properties. These types of fluid are often refers to as viscoelastic fluids. It is very difficult to choose models that will exhibit all the properties of viscolelastic fluids. There exist many viscoelastic models and constitution equations among those is the Voigt Model which consists of a parallel arrangement of a purely viscous element assigned as dashpot and a perfectly elastic body is assigned as a spring. However, the Voigt is a linear viscoelastic rheological model and it is proper only under the condition that non-linear effects are negligible, such as very low strain and stress.

Hydromagnetic or Magnetohydrodynamics (MHD) is the science which deals with the motion of electrically conducting fluid in the presence of magnetic fields. It is the synthesis of two classical sciences, Fluid Mechanics and Electromagnetic field theory. It is well known result in electromagnetic theory that when a conductor moves in a magnetic field, electric currents are induced in it. These current experience a mechanical force, called 'Lorentz Force', due to the presence of magnetic field. This force tends to modify the initial motion of the conductor. Moreover, induced currents generate their own magnetic field which is added on to the primitive magnetic field. Lorentz force, the force exerted on a charged particle q_1 moving with velocity $\vec{V}$ through an electric field $\vec{E}$ and magnetic field $\vec{B}$. The entire electromagnetic force $\vec{F}$on the charged particle is called the Lorentz force (after the Dutch Physicist Hendrik A. Lorentz) and is given by

$$\vec{F} = q_1\vec{E} + q_1(\vec{V}X\vec{B}) \quad (1)$$

The first term is contributed by the electric field. The second term is the magnetic force and has a direction perpendicular to both the velocity and the magnetic field.

Alpher [1] has investigated the heat transfer in magnetohydrodynamic flow between parallel plates. Vidyanidhu and Nigam [2] was discussed the secondary flow in a rotating channel. Pop [3] and Mazumder etal[4] has analyzed the effect of Hall currents on hydromagnetic flow near an accelerated plate. Attia and Kotb[4] has analyzed the MHD flow between two parallel plate with heat transfer. Attia has studied the influences of temperature dependent viscosity on MHD couette flow of dusty fluid with heat transfer. Sanyal and Adhikari [5] have studied the effect of radiation on MHD vertical channel flow. Chand and Kumar [6] has discussed the Soret and hall current effects on heat and mass transfer in MHD flow of viscoelastic fluid past a porous plate in a rotating porous medium in slip flow regime. Hayat etal [6] has analyzed the radiation effects on MHD flow of Maxwell fluid in a channel with porous medium.

The development in the study of dusty viscoelastic fluid flow in a circular channel has been continuously going on because of its unavoidable applications in various fields. Therefore, in this paper, we would try to evolve a mathematical model for a dusty viscoelastic fluid flow in a circular channel. As the magnetic field (both natural and artificial) plays an important role in the motion of fluids, we have considered flows in horizontal circular channel subjected to applied magnetic field so that the results obtained may be applied in different branches of science and technology, industries and problems of medical sciences.

2. FORMULATION OF THE PROBLEM

We assume a linear viscoelastic fluid represented by the Kelvin Model. This model is obtained by thought of the Kelvin element which is subjected to a sudden elongation and the force is then calculated as a function of time. As seen from the mechanical assembly in Figure 1, it is the parallel combination of a spring (elastic element) and a dash pot (viscous element).

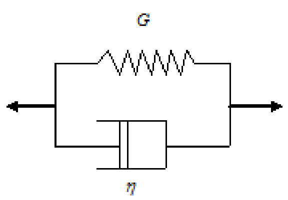

Fig. 1 Kelvin Model

Strain (a) deformation elongation in both elements is same, but due to parallel combination, stress components $\tau_1$and τ_2 added i.e. $\tau = \tau_1 + \tau_2 =$ Total Stress.

Where $\tau_1$and τ_2 are stress components in elastic element and viscous element respectively.

Stress – strain relation is $\tau = Ga + \eta \frac{\partial a}{\partial t}$ (2)

The non-vanishing component of shear strain $'a'$can be expressed in terms of the single nonvanishing component of displacement u by the relation (Kolsky),

$$a = \frac{\partial u}{\partial r} - \frac{u}{r} \quad (3)$$

Newton's second law gives the equation of motion for the viscoelastic plates as (neglecting body forces)

$$\frac{\partial \tau}{\partial r} + \frac{2\tau}{r} = \rho \frac{\partial^2 u}{\partial t^2} \quad (4)$$

Using equation (2) , (3) and (4) we get

$$\tau = G\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right) + \eta\left(\frac{\partial^2 u}{\partial t \partial r} - \frac{1}{r}\frac{\partial u}{\partial t}\right) \quad (5)$$

Consider the flow of a dusty viscoelastic incompressible fluid through a circular channel placed under transverse applied magnetic field taking along the axis of the channel. If we considered that the flow is to be fully developed and symmetric and the velocity of fluid and particle phase are function of radial distance r and time t only. The governing equations of motion of a dusty viscoelastic incompressible fluid are given by:

$$\rho \frac{\partial u}{\partial t} = P(t) + \frac{1}{r}\left(\frac{\partial \tau}{\partial r} + \tau\right) - KN\left(u_p - u\right) - \sigma B_0^2 u \quad (6)$$

Where u(r,t) is axial velocity of fluid, $u_p(r,t)$ is axial velocity of dust particles,Pressure $P(t) = -\frac{dp}{dt}$, K is Stokes resistance coefficient , m is the mass of each particle, N is assumed be constan(the Number density of particles), ρ is the density of fluid , σ is Electrical Conductivity, B_0 is the magnetic Inductor , τ is the shear stress.

And for impurity phase, we have

$$\frac{\partial u_p}{\partial t} = \frac{K}{m}\left(u - u_p\right) \quad (7)$$

With initial and boundary conditions

At $r = 0, r\frac{\partial u}{\partial r} = r\frac{\partial u_p}{\partial r} = 0$ (8)

At $r = 1, u(1) = u_p(1) = 0$ (9)

By substituting equation (5) in equation (6) , we get

$$\rho\frac{\partial u}{\partial t} = -\frac{dp}{dt} + \frac{1}{r}\left[G\left(\frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial r} + \frac{u}{r^2}\right) + \eta\left(\frac{\partial^3 u}{\partial t\partial r^2} + \frac{1}{r^2}\frac{\partial u}{\partial t} + \frac{1}{r}\frac{\partial^2 u}{\partial r\partial t}\right) + G\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right) + \eta\left(\frac{\partial^2 u}{\partial t\partial r} - \frac{1}{r}\frac{\partial u}{\partial t}\right)\right] - KN(u_p - u) - \sigma B_0^2 u \tag{10}$$

Due to oscillatory flow and considering $\lambda = \frac{\mu}{G}$ is called the relaxation time and λ_1 is constant and ω is the frequency of oscillations, so

$$\frac{-1}{\rho}\frac{dp}{dt} = \frac{-1}{\rho}\frac{dp}{dz} = \lambda_1 e^{i\omega t} \tag{11}$$

So, equation (10) can be rewritten as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{dp}{dz} + \frac{1}{r\rho}\left[G\left(\frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial r} + \frac{u}{r^2}\right) + \eta\left(\frac{\partial^3 u}{\partial t\partial r^2} + \frac{1}{r^2}\frac{\partial u}{\partial t} + \frac{1}{r}\frac{\partial^2 u}{\partial r\partial t}\right) + G\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right) + \eta\left(\frac{\partial^2 u}{\partial t\partial r} - \frac{1}{r}\frac{\partial u}{\partial t}\right)\right] - \frac{1}{\rho}KN(u_p - u) - \frac{1}{\rho}\sigma B_0^2 u \tag{12}$$

3. SOLUTION

From equation (11), due to selected form of pressure gradient, the solution will be assumed as

$$u(r,t) = f(r)e^{i\omega t} \tag{13}$$

$$u_p(r,t) = f(r)e^{i\omega t} \tag{14}$$

Considering the dimensionless variables as

$$\bar{r} = \frac{r}{r_0}, \bar{\tau} = \frac{\tau}{r_0}, \bar{u} = \frac{u}{u_0}, \overline{u_p} = \frac{u_p}{u_0}, \bar{t} = \frac{tu_0}{r_0}, \bar{\rho} = \frac{\rho}{\rho u_0^2} \tag{15}$$

Now using equation (15) in equation (12) and equation (7), we get for viscoelastic fluid phase

$$\frac{1}{G}\frac{\partial u}{\partial t} = -\frac{1}{G}\frac{d\rho}{dt} + \frac{1}{Re}\frac{\partial^2 u}{\partial r^2} - \frac{1}{rRe}\frac{\partial u}{\partial r} + \frac{Q}{\eta}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial r}\right) - \frac{R}{Re}(u_p - u) - \frac{H_0}{Re}u \tag{16}$$

And for dust particles phase

$$\frac{\partial u_p}{\partial t} = \frac{1}{R_p}\left(u - u_p\right) \tag{17}$$

Where we defined [1]

$$R_e = \frac{\rho r_0}{G} \quad , = \frac{\eta^2}{\rho G r^2}, R = \frac{KNr_0^2}{G}, H_0 = \frac{\sigma B_0^2 r_0^2}{G}, R_p = \frac{m u_0}{KNr_0} \tag{18}$$

Using equations (13) and (14) , the initial and boundary conditions given by equations (8) and (9) becomes

$$\text{At } r = 0, r\frac{\partial f(0)}{\partial r} = r\frac{\partial g(0)}{\partial r} = 0 \tag{19}$$

$$\text{At } r = 1, f(1) = g(1) = 0 \tag{20}$$

Using equations (13) and (14) in equations (16) and (17), we get

$$\frac{1}{r}\frac{\partial}{\partial t}\left(K_1 r\frac{\partial}{\partial r}\right) - K_2 f - \frac{1}{G}\lambda_1 = 0 \tag{21}$$

$$g = \left(1 + i\omega R_p\right)^{-1} f \tag{22}$$

Where $K_1 = Qi\omega - \frac{1}{Re}$

$$K_2 = \frac{R}{Re} - i\omega + \frac{H_a}{Re} - \frac{R}{Re(1+i\omega R_p)} \tag{23}$$

4. CONCLUSION

The effect of magnetic field and Viscoelastic parameter is to be studied in incompressible viscoelastic fluid in a circular pipe. It is observed fro equations (22) and (23), the velocity of both the fluid and dust particle is not effected at low value of magnetic field and viscoelastic parameter i.e. $H_0 = \frac{\sigma B_0^2 r_0^2}{G}$ and $Q = \frac{\eta^2}{\rho G r^2}$. When there is increase in magnetic field and viscoelastic parameters, the horizontal velocity of fluid and particle decreases rapidly. This is due to that dust particles experiences an additional force while moving in a magnetic field. Here the boundaries of the circular pipe are having radius between zero and one. So, it can be concluded that thickness of the boundary layer is much larger for higher value of Viscoelastic parameter than that of magnetic field parameter. The results are same as given by Aiyesimi Y.M. and Lawal O.W(2013) for analysis of magnetohydrodynamics flow of a dusty viscoelastic fluid through a horizontal circular channel for Maxwell Model.[9]

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