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Bayesian Analysis of Weibull Pareto Distribution Using R Software

Kawsar Fatima[1], S.P Ahmad[2]

[1,2](*Department of Statistics, University of Kashmir, Srinagar, India*)

ABSTRACT

The main objective of our research problem is to study the Bayes estimation of the unknown parameter of a three parameter Weibull Pareto distribution (WPD). The prior distribution used here is the non-informative Jeffery's prior; Extension of Jeffery's prior and quasi prior. Bayes estimators are derived under squared error loss function, Entropy loss function and Precautionary loss function which is asymmetric in nature. Mean square error simulations are performed to compare the performances of these Bayes estimates under different situations. Finally, we summarize the result and give the conclusion of this study.

Keywords: Baye's estimation, Loss functions, Maximum Likelihood Estimator, Priors, Weibull Pareto distribution.

I. INTRODUCTION

A new three-parameter distribution, called the Weibull Pareto distribution (WP) has been introduced recently by Suleman Nasiru (2015). The three parameters WP has the following density function

$$f(x;\delta,\beta,\theta)=\frac{\beta\delta}{\theta}\left(\frac{x}{\theta}\right)^{(\beta-1)}e^{-\delta\left(\frac{x}{\theta}\right)^{\beta}} \quad ;\ x>0 \text{ and } \theta,\beta,\delta>0, \quad . \tag{1.1}$$

and the cumulative distribution function cdf of the WP distribution is

$$F(x;\delta,\beta,\theta)=1-e^{-\delta\left(\frac{x}{\theta}\right)^{\beta}} \quad ;\ x>0 \text{ and } \theta,\beta,\delta>0 \quad . \tag{1.2}$$

For β *and* $\theta=1$, it represents the one parameter exponential distribution , for β *and* $\delta=1$, it represents the one parameter inverted exponential distribution , for δ *and* $\theta=1$, it represents the one parameter Weibull distribution , for $\delta=1$, it represents the two parameter inverted Weibull distribution , for $\theta=1$, it represents the two parameter Weibull distribution and for $\beta=1$, it represents the two parameter inverted exponential distribution.

The Weibull distribution is one of the most widely used distributions for analyzing lifetime data. It is found to be useful in diverse fields ranging from engineering to medical sciences (see Lawless (2002), Martz and Waller (1982)). The Weibull family is a generalization of the exponential family and can model data exhibiting monotone hazard rate behavior, i.e. it can accommodate three types of failure rates, namely increasing, decreasing and constant. The Pareto distribution was introduced (Pareto, 1896) as a model for the distribution of income. In addition to economics, its models in several different forms are now being used in a wide range of fields such as insurance, business, engineering, survival analysis, reliability and life testing. Alzaatreh *et al*. (2013) developed the Weibull-Pareto distribution while Bourguignon et al. (2014), introduced the Weibull-G family of distributions. Ahmad and Kaisar (2013) considered the estimation of the scale parameter of two parameter Weibull distribution with known shape. They obtained Baye's estimator of Weibull distribution by using Jeffrey's and extension of Jeffrey's prior under linear exponential loss function and symmetric loss functions. Afaq et al. (2014) considered the estimation of the parameters of Lomax distribution using Jeffery's and extension of Jeffery's prior under different loss functions. They also compared the classical method with Bayesian method by using mean square error through simulation study with varying sample sizes. Dow, James, (2015) studied the Bayesian Inference of the Weibull-Pareto distribution.

The aim of this paper is to propose the different methods of estimation of the parameters of the WPD. In the next section, we obtain the MLE of the unknown parameter δ, in WPD when the parameters β *and* θ are known. We also discuss the procedures to obtain the Bayes estimators for the unknown parameters using Jeffery's prior, extension of Jeffery's and Quasi prior under entropy loss, square error and precautionary loss function.

II. ESTIMATION OF THE UNKNOWN PARAMETER δ WHEN β AND θ ARE KNOWN

Let us consider a random sample $\underline{x}=(x_1,x_2,...,x_n)$ of size n from the Weibul Pareto distribution (WPD). Then the likelihood function for the given sample observation is

$$L(\delta,\beta,\theta)=\frac{\beta^n\delta^n}{\theta^n}\prod_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta-1}e^{-\delta\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta}}. \tag{2.1}$$

The log-likelihood function is

$$\ln L(\delta,\beta,\theta)=n\ln\beta+n\ln\delta-n\ln\theta-\delta\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta}+(\beta-1)\sum_{i=1}^{n}\ln\left(\frac{x_i}{\theta}\right). \tag{2.2}$$

As parameters θ and β are assumed to be known, the ML estimator of unknown parameter δ is obtained by solving the

$$\frac{\partial}{\partial\delta}\ln L(\delta,\beta,\theta)=\frac{n}{\delta}-\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta}=0 \quad\Rightarrow\quad \hat{\delta}=\frac{n}{\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta}}. \tag{2.3}$$

III. BAYESIAN ESTIMATION

The foundation of Bayesian statistics is Bayes theorem. Bayes theorem states that posterior density is proportional to the product of prior and likelihood function i.e.,

$$g(\delta\,|\,x)\propto L(x;\delta,\beta,\theta)\;g(\delta) \quad .$$

In the Bayesian terminology, $g(\delta)$ is a prior density of δ, which tells us what is known about δ without knowledge of data. The density $L(x;\delta,\beta,\theta)$ is likelihood function of δ, which represents the contribution of

x (data) to knowledge about δ (e.g., Berger, 1985 and Zellener, 1971). Finally, $g(\delta|x)$ is the posterior density, which tells us what is known about δ given knowledge of data x .Posterior estimates of δ can also be obtained from the posterior density $g(\delta|x)$. In this paper we now derive the Baye's estimator of the unknown parameter δ in WPD when the parameters, θ *and* β are known. In this paper, we consider three different priors and three different loss functions.

(1) Jeffery's prior: Jeffery's (1946) proposed a formal rule for obtaining a non-informative prior as

$$g_1(\delta) \propto \frac{1}{\delta}.$$

(2) Extension of Jeffery's Prior: The extended Jeffrey's prior proposed by Al-Kutubi (2005), is given as:

$$g_2(\theta) \propto \frac{1}{\delta^{2c_1}} \quad .$$

(3) Quasi Prior: when there is no information about the parameter δ , one may use the quasi density as given by:

$$g_3(\delta) \propto \frac{1}{\delta^d}, \qquad \delta > 0, \quad d > 0$$

The quasi prior leads to diffuse prior when d=0 and to a non informative prior for a case when d=1.

3.1 Baye's Estimator under $g_1(\delta)$

Under $g_1(\delta)$, using (2.1), the posterior distribution of δ is given by

$$g_1(\delta \mid x) \propto \frac{\beta^n \delta^n}{\theta^n} \prod_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta-1} e^{-\delta \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}} \quad \frac{1}{\delta}$$

$$g_1(\delta \mid x) = K\delta^{n-1} e^{-\delta T} \quad ,$$

where K is independent of δ and $T = \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}$

$$\Rightarrow \quad K^{-1} = \frac{\Gamma n}{T^n} \quad .$$

Hence the posterior distribution function of δ is given as

$$g_1(\delta \mid x) = \frac{T^n}{\Gamma n} \delta^{n-1} e^{-\delta T} \quad ; \quad \delta > 0 \quad . \qquad (3.1)$$

3.1.1 Estimation under SELF

By using squared error loss function $l(\hat{\delta}, \delta) = c(\hat{\delta} - \delta)^2$, for some constant c the risk function is given by

$$R(\hat{\delta}) = \int_0^\infty c(\hat{\delta}-\delta)^2 \frac{T^n}{\Gamma n}\,\delta^{n-1}e^{-\delta T}d\delta$$

$$R(\hat{\delta}) \;= \frac{cT^n}{\Gamma(n)}\left(\hat{\delta}^2\int_0^\infty e^{-\delta T}\delta^{n-1}d\delta + \int_0^\infty e^{-\delta T}\delta^{n+2-1}d\delta - 2\hat{\delta}\int_0^\infty e^{-\delta T}\delta^{n+1-1}d\delta\right)$$

$$R(\hat{\delta}) = c\hat{\delta}^2 + \frac{n(n+1)c}{T^2} - \frac{2cn\hat{\delta}}{T}.$$

Now solving $\frac{\partial}{\partial\hat{\delta}}R(\hat{\delta}) = 0$, we obtain the Bayes estimator as

$$\hat{\delta}_{1JS} = \frac{n}{T}\ . \tag{3.2}$$

3.1.2 Estimation under Entropy Loss Function

By using entropy loss function $L(\lambda)=b[\lambda-\log(\lambda)-1]$ for some constant b the risk function is given by

$$R(\hat{\delta}) = \int_0^\infty b\left(\frac{\hat{\delta}}{\delta} - \log\left(\frac{\hat{\delta}}{\delta}\right) - 1\right)\frac{T^n}{\Gamma n}e^{-\delta T}\delta^{n-1}\,d\delta$$

$$R(\hat{\delta}) = \frac{T^n b}{\Gamma n}\left[\hat{\delta}\int_0^\infty \delta^{n-1-1}e^{-\delta T}d\delta - \ln(\hat{\delta})\int_0^\infty \delta^{n-1}e^{-\delta T}d\delta + \int_0^\infty \ln(\delta)\,\delta^{n-1}e^{-\delta T}d\delta - \int_0^\infty \delta^{n-1}e^{-\delta T}d\delta\right]$$

$$R(\hat{\delta}) = b\left[\frac{T\hat{\delta}}{(n-1)} - \ln(\hat{\delta}) + \frac{\Gamma'(n)}{\Gamma(n)} - 1\right].$$

Now solving $\frac{\partial}{\partial\hat{\delta}}R\left(\hat{\delta}\right) = 0$, we obtain the Baye's estimator as

$$\hat{\delta}_{1JE} = \frac{(n-1)}{T}\ . \tag{3.3}$$

3.1.3 Estimation under precautionary loss function

By using precautionary loss function $l(\hat{\delta},\delta) = \frac{(\hat{\delta}-\delta)^2}{\hat{\delta}}$, the Risk function is given by

$$R(\hat{\delta}) = \int_0^\infty \frac{(\hat{\delta}-\delta)^2}{\hat{\delta}}\frac{T^n}{\Gamma n}\,\delta^{n-1}e^{-\delta T}d\delta$$

$$R(\hat{\delta}) \;= \frac{T^n}{\Gamma(n)\hat{\delta}}\left(\hat{\delta}^2\int_0^\infty e^{-\delta T}\delta^{n-1}d\delta + \int_0^\infty e^{-\delta T}\delta^{n+2-1}d\delta - 2\hat{\delta}\int_0^\infty e^{-\delta T}\delta^{n+1-1}d\delta\right)$$

$$R(\hat{\delta}) \;= \hat{\delta} + \frac{n(n+1)}{T^2\hat{\delta}} - \frac{2n}{T}.$$

Now solving $\frac{\partial}{\partial \hat{\delta}} R(\hat{\delta}) = 0$, we obtain the Bayes estimator as

$$\hat{\delta}_{1JP} = \frac{[n(n+1)]^{\frac{1}{2}}}{T} \quad . \tag{3.4}$$

3.2 Baye's Estimator under $g_2(\delta)$

Under $g_2(\delta)$, using (2.1), the posterior distribution of δ is given

by $g_2(\delta \mid x) \propto \frac{\beta^n \delta^n}{\theta^n} \prod_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta-1} e^{-\delta \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}} \frac{1}{\delta^{2c_1}}$

$$g_2(\delta \mid x) = K \delta^{n-2c_1} e^{-\delta \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}} ,$$

where K is independent of δ and $T = \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}$.

$$\Rightarrow \qquad K^{-1} = \frac{\Gamma(n-2c_1+1)}{T^{n-2c_1+1}} \quad .$$

Hence the posterior distribution function of δ is given as

$$g_2(\delta \mid x) = \frac{T^{n-2c_1+1}}{\Gamma(n-2c_1+1)} \delta^{n-2c_1} e^{-\delta T} \; ; \; \delta > 0 \quad . \tag{3.5}$$

3.2.1 Estimation under SELF

By using squared error loss function $l(\hat{\delta}, \delta) = c(\hat{\delta} - \delta)^2$, the Risk function is given by

$$R(\hat{\delta}) = \int_0^{\infty} c(\hat{\delta} - \delta)^2 \frac{T^{n-2c_1+1}}{\Gamma(n-2c_1+1)} \delta^{n-2c_1} e^{-\delta T} d\delta$$

$$R(\hat{\delta}) = \frac{cT^{n-2c_1+1}}{\Gamma(n-2c_1+1)} \left(\hat{\delta}^2 \int_0^{\infty} e^{-\delta T} \delta^{n-2c_1} d\delta + \int_0^{\infty} e^{-\delta T} \delta^{n-2c_1+2} d\delta - 2\hat{\delta} \int_0^{\infty} e^{-\delta T} \delta^{n-2c_1+1} d\delta \right).$$

$$R(\hat{\delta}) = c\hat{\delta}^2 + \frac{c(n-2c_1+2)(n-2c_1+1)}{T^2} - \frac{2c(n-2c_1+1)\hat{\delta}}{T} .$$

Now solving $\frac{\partial}{\partial \hat{\delta}} R(\hat{\delta}) = 0$, we obtain the Bayes estimator as

$$\hat{\delta}_{2EJS} = \frac{(n-2c_1+1)}{T} \quad . \tag{3.6}$$

Remark 1.1: Replacing $c_1 = 1/2$ in (3.6) we get the same Bayes estimators as obtained in (3.2) corresponding to the Jeffrey's prior and replace $c_1 = 3/2$ we get the Hartigan' s prior.

3.2.2 Estimation under Entropy Loss Function

By using entropy loss function $L(\lambda)=b[\lambda-\log(\lambda)-1]$ for some constant b the risk function is given by

$$R(\hat{\delta})=\int_0^\infty b\left(\frac{\hat{\delta}}{\delta}-\log\left(\frac{\hat{\delta}}{\delta}\right)-1\right)\frac{T^{n-2c_1+1}}{\Gamma(n-2c_1+1)}e^{-\delta T}\delta^{n-2c_1}\,d\delta$$

$$R(\hat{\delta})=\frac{bT^{n-2c_1+1}}{\Gamma(n-2c_1+1)}\left[\begin{array}{r}\hat{\delta}\int_0^\infty \delta^{n-2c_1-1}e^{-\delta T}d\delta-\ln(\hat{\delta})\int_0^\infty \delta^{n-2c_1+1-1}e^{-\delta T}d\delta \\ +\int_0^\infty \ln(\delta)\,\delta^{n-2c_1+1-1}e^{-\delta T}d\delta-\int_0^\infty \delta^{n-2c_1+1-1}e^{-\delta T}d\delta\end{array}\right]$$

$$R(\hat{\delta})=b\left[\frac{T\hat{\delta}}{(n-2c_1)}-\ln(\hat{\delta})+\frac{\Gamma'(n-2c_1+1)}{\Gamma(n-2c_1+1)}-1\right].$$

Now solving $\frac{\partial}{\partial\hat{\delta}}R(\hat{\delta})=0$, we obtain the Baye's estimator as

$$\hat{\delta}_{2EJE}=\frac{(n-2c_1)}{T}\;. \tag{3.7}$$

Remark 1.2: replacing $c_1=1/2$ in (3.7) we get the same Bayes estimators as obtained in (3.3) corresponding to the Jeffrey's prior and replace $c_1=3/2$ we get the Hartigan' s prior.

3.2.3 Estimation under precautionary loss function

By using precautionary loss function $l(\hat{\delta},\delta)=\frac{(\hat{\delta}-\delta)^2}{\hat{\delta}}$, the Risk function is given by

$$R(\hat{\delta})=\int_0^\infty\frac{(\hat{\delta}-\delta)^2}{\hat{\delta}}\frac{T^{n-2c_1+1}}{\Gamma(n-2c_1+1)}\delta^{n-2c_1}e^{-\delta T}d\delta$$

$$R(\hat{\delta})=\frac{T^{n-2c_1+1}}{\Gamma(n-2c_1+1)\hat{\delta}}\left(\hat{\delta}^2\int_0^\infty\delta^{n-2c_1+1-1}e^{-\delta T}d\delta+\int_0^\infty e^{-\delta T}\delta^{n-2c_1+3-1_1}d\delta-2\hat{\delta}\int_0^\infty e^{-\delta T}\delta^{n-2c_1+2-1_1}d\delta\right)$$

$$R(\hat{\delta})=\hat{\delta}+\frac{(n-2c_1+2)(n-2c_1+1)}{T^2\hat{\delta}}-\frac{2(n-2c_1+1)}{T}.$$

Now solving $\frac{\partial}{\partial\hat{\delta}}R(\hat{\delta})=0$, we obtain the Bayes estimator as

$$\hat{\delta}_{2EJP}=\frac{[(n-2c_1+2)(n-2c_1+1)]^{\frac{1}{2}}}{T} \tag{3.8}$$

Remark 1.3: Replacing $c_1 = 1/2$ in (3.8) we get the same Bayes estimators as obtained in (3.4) corresponding to the Jeffrey's prior and replace $c_1 = 3/2$ we get the Hartigan' s prior.

3.3 Baye's Estimator under $g_3(\delta)$

Under $g_3(\delta)$, using (2.1), the posterior distribution of δ is given by

$$g_3(\delta \mid x) \propto \frac{\beta^n \delta^n}{\theta^n} \prod_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta-1} e^{-\delta \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}} \quad \frac{1}{\delta^d}$$

$$g_3(\delta \mid x) = K\,\delta^{n-d} e^{-\delta \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}} \quad ,$$

where K is independent of δ and $T = \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta} \Rightarrow K^{-1} = \frac{\Gamma(n-d+1)}{T^{n-d+1}}$.

Hence the posterior distribution function of δ is given as

$$g_3(\delta \mid x) = \frac{T^{n-d+1}}{\Gamma(n-d+1)} \delta^{n-d} e^{-\delta T} \;; \delta > 0 \tag{3.9}$$

3.3.1 Estimation under SELF

By using squared error loss function $l(\hat{\delta}, \delta) = c(\hat{\delta} - \delta)^2$, the Risk function is given by

$$R(\hat{\delta}) = \int_0^{\infty} c(\hat{\delta} - \delta)^2 \frac{T^{n-d+1}}{\Gamma(n-d+1)} \delta^{n-d} e^{-\delta T} d\delta$$

$$R(\hat{\delta}) = \frac{cT^{n-d+1}}{\Gamma(n-d+1)} \left(\hat{\delta}^2 \int_0^{\infty} e^{-\delta T} \delta^{n-d} d\delta + \int_0^{\infty} e^{-\delta T} \delta^{n-d+2} d\delta - 2\hat{\delta} \int_0^{\infty} e^{-\delta T} \delta^{n-d+1} d\delta \right).$$

$$R(\hat{\delta}) = c\hat{\delta}^2 + \frac{c(n-d+2)(n-d+1)}{T^2} - \frac{2c(n-d+1)\hat{\delta}}{T}.$$

Now solving $\frac{\partial}{\partial \hat{\delta}} R(\hat{\delta}) = 0$, we obtain the Bayes estimator as

$$\hat{\delta}_{3QS} = \frac{(n-d+1)}{T} \quad . \tag{3.10}$$

3.3.2 Estimation under Entropy Loss Function

By using entropy loss function $L(\lambda) = b[\lambda - \log(\lambda) - 1]$ for some constant b the risk function is given by

$$R(\hat{\delta}) = \int_0^{\infty} b\left(\frac{\hat{\delta}}{\delta} - \log\left(\frac{\hat{\delta}}{\delta}\right) - 1 \right) \frac{T^{n-d+1}}{\Gamma(n-d+1)} e^{-\delta T} \delta^{n-d} \, d\delta$$

$$R(\hat{\delta})=\frac{bT^{n-d+1}}{\Gamma(n-d+1)}\left[\hat{\delta}\int_0^\infty \delta^{n-d-1}e^{-\delta T}d\delta-\ln(\hat{\delta})\int_0^\infty \delta^{n-d+1-1}e^{-\delta T}d\delta+\int_0^\infty \ln(\delta)\,\delta^{n-d+1-1}e^{-\delta T}d\delta-\int_0^\infty \delta^{n-d+1-1}e^{-\delta T}d\delta\right]$$

$$R(\hat{\delta})=b\left[\frac{T\hat{\delta}}{(n-d)}-\ln(\hat{\delta})+\frac{\Gamma'(n-d+1)}{\Gamma(n-d+1)}-1\right].$$

Now solving $\frac{\partial}{\partial\hat{\delta}}R(\hat{\delta})=0$, we obtain the Baye's estimator as

$$\hat{\delta}_{3QE}=\frac{(n-d)}{T}. \tag{3.11}$$

3.3.3 Estimation under precautionary loss function

By using precautionary loss function $l(\hat{\delta},\delta)=\frac{(\hat{\delta}-\delta)^2}{\hat{\delta}}$, the Risk function is given by

$$R(\hat{\delta})=\int_0^\infty \frac{(\hat{\delta}-\delta)^2}{\hat{\delta}}\frac{T^{n-d+1}}{\Gamma(n-d+1)}\delta^{n-d}e^{-\delta T}d\delta$$

$$R(\hat{\delta})=\frac{T^{n-d+1}}{\Gamma(n-d+1)\hat{\delta}}\left(\hat{\delta}^2\int_0^\infty \delta^{n-d+1-1}e^{-\delta T}d\delta+\int_0^\infty e^{-\delta T}\delta^{n-d+3-1_1}d\delta-2\hat{\delta}\int_0^\infty e^{-\delta T}\delta^{n-d+2-1_1}d\delta\right)$$

$$R(\hat{\delta})=\hat{\delta}+\frac{(n-d+2)(n-d+1)}{T^2\hat{\delta}}-\frac{2(n-d+1)}{T}.$$

Now solving $\frac{\partial}{\partial\hat{\delta}}R(\hat{\delta})=0$, we obtain the Bayes estimator as

$$\hat{\delta}_{3QP}=\frac{[(n-d+2)(n-d+1)]^{\frac{1}{2}}}{T}. \tag{3.12}$$

IV. SIMULATION STUDY

In our simulation study, we choose a sample size of n=25, 50 and 100 to represent small, medium and large data set. The parameter δ is estimated for Weibull Pareto distribution by using the Bayesian method of estimation under Jeffrey's, extension of Jeffrey's prior and Quasi prior by using different loss functions. For the parameter δ we have considered δ = 0.5, 1.0 and 1.5. The parameters θ and β has been fixed at θ =0.5 and β =1.5. The value of Jeffrey's extension were c_1= 0.4 and the hyper parameter d=0.3. This was iterated 10000 times and the shape parameter for each method was calculated. A simulation study was conducted R-software to examine and compare the performance of the estimates for different sample sizes with different values of loss functions. The results are presented in tables for different selections of the parameters. It is clear from Table 1-3, the comparison of mean square error under different loss functions using non-informative priors has been made through which we conclude that within each prior entropy loss function provides less mean square error so it is

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more suitable for the Weibull Pareto distribution and amongst priors Jeffrey prior is more preferable as compared to other priors which are provided here because under this prior mean square error is small.

Table 1: MSE for $\hat{\delta}$ under Jeffery prior using different loss functions

n	θ	β	δ	$\hat{\delta}_{ML}$	$\hat{\delta}_{SL}$	$\hat{\delta}_{EL}$	$\hat{\delta}_{PL}$
25	0.5	1.5	0.5	0.036934	0.036934	0.029019	0.041373
			1.0	0.277250	0.277250	0.224239	0.306084
			1.5	0.142988	0.142988	0.167373	0.132884
50	0.5	1.5	0.5	0.008022	0.008022	0.009132	0.0075285
			1.0	0.016252	0.016252	0.016074	0.016648
			1.5	0.046349	0.046349	0.044440	0.048012
100	0.5	1.5	0.5	0.005770	0.005770	0.006238	0.005552
			1.0	0.010151	0.010151	0.009824	0.010392
			1.5	0.033991	0.033991	0.036217	0.033033

Table 2: MSE for $\hat{\delta}$ under Extension of Jeffery's prior using different loss functions

n	θ	β	δ	C_1	$\hat{\delta}_{ML}$	$\hat{\delta}_{SL}$	$\hat{\delta}_{EL}$	$\hat{\delta}_{PL}$
25	0.5	1.5	0.5	0.4	0.037018	0.0387706	0.0305741	0.043349
			1.0	0.4	0.277641	0.289083	0.234673	0.318610
			1.5	0.4	0.143777	0.139538	0.162859	0.129960
50	0.5	1.5	0.5	0.4	0.0080410	0.007837	0.008916	0.007359
			1.0	0.4	0.016316	0.016451	0.016108	0.016928
			1.5	0.4	0.046526	0.047137	0.044846	0.048990
100	0.5	1.5	0.5	0.4	0.005777	0.005688	0.006148	0.005474
			1.0	0.4	0.010170	0.010261	0.009892	0.010522
			1.5	0.4	0.034049	0.0336531	0.035797	0.032736

Table 3: MSE for $\hat{\delta}$ under Quasi prior using different loss functions

n	θ	β	δ	d	$\hat{\delta}_{ML}$	$\hat{\delta}_{SL}$	$\hat{\delta}_{EL}$	$\hat{\delta}_{PL}$
25	0.5	1.5	0.5	0.3	0.037229	0.043605	0.034706	0.051607
			1.0	0.3	0.278618	0.319885	0.261981	0.370433
			1.5	0.3	0.145749	0.131845	0.152506	0.119313
50	0.5	1.5	0.5	0.3	0.008087	0.007403	0.008403	0.006719
			1.0	0.3	0.016477	0.017091	0.016337	0.018277
			1.5	0.3	0.046968	0.049443	0.046194	0.053383
100	0.5	1.5	0.5	0.3	0.005793	0.005490	0.005929	0.005170
			1.0	0.3	0.010219	0.010571	0.010099	0.011097
			1.5	0.3	0.034194	0.032878	0.034819	0.031625

From these tables we conclude that Entropy loss is best among these priors as it gives the smallest values of estimates in most cases

V. CONCLUSION

In this paper, we have addressed the problem of Bayesian estimation for the Weibull Pareto distribution, under three different loss functions and that of Maximum Likelihood Estimation. From the results, we observe that in most cases, Bayesian Estimator under entropy Loss function has the smallest Mean Squared Error values under Jeffrey's prior and its extension prior as well as the Quasi prior. Further as we increase sample size mean square error comes down.

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