International Journal of Advance Research in Science and Engineering
Volume No.07, Special Issue No.04, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

A NEW TWO PARAMETRIC FUZZY DIVERGENCE MEASURE AND ITS PROPERTIES

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ABSTRACT

In the present paper, we define a two parametric fuzzy divergence measure and also verify some of the important properties. With the help of R software, we establish the results.

Keywords*: *Fuzzy directed divergence Fuzzy information and Fuzzy set.

I. INTRODUCTION

Fuzzy information has wide applications in different fields as Management and Decision making, Engineering, Computer Sciences, Pattern Recognition, Medicine, Social Sciences, Fuzzy Aircraft control, Life Sciences etc. With these numerous applications we are motivated to study divergence measures for fuzzy set theory to optimize the fuzziness and to make the available information more reliable.

For measuring the degree of dissimilarity between two fuzzy sets various measures, known as fuzzy divergence measures, have been introduced. Let us consider two fuzzy sets *A* and *B* with their respective membership functions as $\mu_A(x_i)$ and $\mu_B(x_i)$ for $i = 1, 2, ..., n$. Bhandari et al. [1] defined the fuzzy directed divergence measure corresponding to Kullback and Leibler [2] measure as:

$$I(A:B) = \sum_{i=1}^{n}\left[\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)} + (1-\mu_A(x_i))\log\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right].$$

Further, Kapur [3] proposed the divergence measure corresponding to Havrda-Charvat [4] measure as:

$$I_\alpha(A,B) = \frac{1}{\alpha-1}\sum\left[\mu_A^\alpha\,\mu_B^{1-\alpha} + (1-\mu_A)^\alpha(1-\mu_B)^{1-\alpha} - 1\right], \alpha \neq 1,\ \alpha > 0.$$

Later on, several other researchers gave different fuzzy directed divergence measures which include Bajaj and Hooda [5], Verma and Sharma [6], Tomar and Ohlan [7], Saima Manzoor Sofi et al. [8] etc.

This paper comprises of four sections. Section 1 gives the introduction with the basic concepts. Section 2 corresponds a new measure of fuzzy divergence and its properties are discussed in the section 3. At the end, in section 4 the conclusion of the paper is given.

II. NEW FUZZY DIVERGENCE MEASURE

Consider the following fuzzy directed divergence measure between two fuzzy sets *A* and *B* corresponding to the generalized fuzzy information measure of order α and type β given by Safeena Peerzada et al. [9]

$$I(A,B) = -\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)} + (1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_{\mathbf{B}}(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$\alpha \geq 0, \alpha \neq 1 \;\&\; 0 < \beta \leq 1 \qquad (1)$$

and fuzzy symmetric divergence measure as:

$$J(A,B) = I(A,B) + I(B,A)$$

For (1) to be a valid fuzzy directed divergence measure it should satisfy the following fundamental properties:

(i) $I(A,B) \geq 0$.

(ii) $I(A,B) = 0$ if $\mu_A(x_i) = \mu_B(x_i)$.

(iii) $I(A,B) \neq I(B,A)$.

(iv) There should be no change in $I(A,B)$ if we replace $\mu_A(x_i)$ by $1-\mu_A(x_i)$ and $\mu_B(x_i)$ by $1-\mu_B(x_i)$.

(v) $I(A,B)$ is convex i.e., $\frac{\partial^2 I(A,B)}{\partial\mu_A^2(x_i)} > 0$ & $\frac{\partial^2 I(A,B)}{\partial\mu_B^2(x_i)} > 0$ for $\alpha \geq 0, \alpha \neq 1 \;\&\; 0 < \beta \leq 1$.

Example: Considering two arbitrary fuzzy sets *A* & *B*, the above properties are verified in the following tables.

Table 1: Property 1

$\mu_A(x_i)$	$\mu_B(x_i)$	α	β	$I(A,B)$
0.42	0.66	0.3	0.7	0.015804
0.11	0.18			
0.64	0.32			
0.91	0.99			
0.21	0.70			

Table 2: Property 2

$\mu_A(x_i)$	$\mu_B(x_i)$	α	β	$I(A,B)$
0.42	0.42	0.3	0.7	0.0
0.11	0.11			
0.64	0.64			
0.91	0.91			
0.32	0.32			

From Table (1) & (2), we see that $I(A,B)$ is non-negative (i.e., $I(A,B)\geq 0$) & $I(A,B)=0$ for $\mu_A(x_i)=\mu_B(x_i)$ respectively.

Table 3: Property 3

$\mu_A(x_i)$	$\mu_B(x_i)$	α	β	$I(A,B)$	$I(B,A)$
0.42	0.66	0.3	0.2	0.015804	0.0161194
0.11	0.18				
0.64	0.32				
0.91	0.99				
0.21	0.70				

It is clear from Table (3) that $I(A,B)\neq I(B,A)$.

Table 4: Property 4

$\mu_A(x_i)$	$1-\mu_A(x_i)$	$\mu_B(x_i)$	$1-\mu_B(x_i)$	α	β	$I(A,B)$
0.42	0.58	0.66	0.34	0.3	0.7	0.02288515
0.11	0.89	0.18	0.82			
0.64	0.36	0.32	0.68			
0.91	0.09	0.99	0.01			
0.79	0.21	0.70	0.30			

It is obvious from the above Table (4) that $I(A,B)$ remains same if we change $\mu_A(x_i)$ by $1-\mu_A(x_i)$ and $\mu_B(x_i)$ by $1-\mu_B(x_i)$.

Table 5: Property 5

$\mu_A(x_i)$	$\mu_B(x_i)$	α	β	$\frac{\partial^2 I(A,B)}{\partial\mu_A^2(x_i)}$	$\frac{\partial^2 I(A,B)}{\partial\mu_B^2(x_i)}$
0.32	0.66	0.32	0.65	0.3354294	0.4116112
0.71	0.32			0.6993515	0.3214891

0.58	0.53			0.3757758	0.2835604
0.42	0.18			0.3224071	0. 5009692
0.88	0.74			1.5461909	0. 3280825

Table 5 clearly shows that $I(A,B)$ is convex i.e., $\frac{\partial^2 I(A,B)}{\partial\mu_A^2(x_i)}>0$ & $\frac{\partial^2 I(A,B)}{\partial\mu_B^2(x_i)}>0$ for $\alpha\geq 0, \alpha\neq 1\&\ 0<\beta\leq 1$.

Thus, by using R software, we observe that the proposed measure satisfies all the fundamental properties. Hence, we conclude that (1) is a valid fuzzy divergence measure.

In particular, we have

1. For $\alpha=0$ & $\beta=1, I(A,B)=0.$
2. For $\beta=0, I(A,B)=0.$

III. SOME MORE PROPERTIES OF THE FUZZY DIVERGENCE MEASURE:

The fuzzy divergence measure defined above satisfies some more properties as given below:

(i) $I(A\cup B,A)+I(A\cap B,A)=I(B,A)$.

Proof: Let $X_1 \& X_2$ be two fuzzy sets such that $X_1=\{x/x\in X_i, \mu_A(x_i)\geq\mu_B(x_i)\}$ & $X_2=\{x/x\in X_i, \mu_B(x_i)>\mu_A(x_i)\}$.

In set X_1, we have

$$\mu_{A\cup B}(x)=\max\{\mu_A(x_i),\mu_B(x_i)\}=\mu_A(x_i)\ \&\ \mu_{A\cap B}(x)=\min\{\mu_A(x_i),\mu_B(x_i)\}=\mu_B(x_i).$$

In set X_2, we have

$$\mu_{A\cup B}(x)=\max\{\mu_A(x_i),\mu_B(x_i)\}=\mu_B(x_i)\ \&\ \mu_{A\cap B}(x)=\min\{\mu_A(x_i),\mu_B(x_i)\}=\mu_A(x_i).$$

Now,

$$I(A,B)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_{\mathbf{B}}(x_i))^{1-\beta(1-\alpha)}\right\}\right].$$

Consider $I(A\cup B,A)+I(A\cap B,A)=$

IJARSE
ISSN: 2319-8354

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{A\cup B}(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{A\cup B}(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\right]+$$

$$\left[-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{A\cap B}(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{A\cap B}(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\right]\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\frac{1}{n}\left[\begin{array}{l}\sum_{X_1}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}+\\ \sum_{X_2}\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\end{array}\right]$$

$$-\frac{\beta}{1-\alpha}\log_D\frac{1}{n}\left[\begin{array}{l}\sum_{X_1}\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}+\\ \sum_{X_2}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\end{array}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\frac{1}{n}\left[\sum_{X_2}\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$-\frac{\beta}{1-\alpha}\log_D\frac{1}{n}\left[\sum_{X_1}\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\frac{1}{n}\left[\sum_{X_i}\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$=I(B,A)$.

This establishes (i).

(ii) $I(A\cup B,C)+I(A\cap B,C)=I(A,C)+I(B,C)$.

Proof: L.H.S. =

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{A\cup B}(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{A\cup B}(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}\right]+$$

$$\left[-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{A\cap B}(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{A\cap B}(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}\right]\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\frac{1}{n}\left[\begin{array}{l}\sum_{X_1}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}+\\ \sum_{X_2}\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}\end{array}\right]$$

$$-\frac{\beta}{1-\alpha}\log_D \frac{1}{n}\left[\begin{array}{l}\sum_{X_1}\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}+\\ \sum_{X_2}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}\end{array}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D \frac{1}{n}\left[\sum_{i=1}^{n}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}\right]+$$

$$\left[-\frac{\beta}{1-\alpha}\log_D \frac{1}{n}\left[\sum_{i=1}^{n}\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}\right]\right]$$

$$=I(A,C)+I(B,C)$$

(iii) $I(\overline{A\cup B},\overline{A\cap B})=I(\overline{A}\cap\overline{B},\overline{A}\cup\overline{B})$

Proof: Consider $I(\overline{A\cup B},\overline{A\cap B})=$

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{\overline{A\cup B}}(x_i)^{\beta(1-\alpha)}\mu_{\overline{A\cap B}}(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{\overline{A\cup B}}(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{A\cap B}}(x_i))^{1-\beta(1-\alpha)}\right\}\right] \quad (2)$$

From R.H.S., we have

$$I(\overline{A}\cap\overline{B},\overline{A}\cup\overline{B})=$$

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{\overline{A}\cap\overline{B}}(x_i)^{\beta(1-\alpha)}\mu_{\overline{A}\cup\overline{B}}(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{\overline{A}\cap\overline{B}}(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{A}\cup\overline{B}}(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{\overline{A\cup B}}(x_i)^{\beta(1-\alpha)}\mu_{\overline{A\cap B}}(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{\overline{A\cup B}}(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{A\cap B}}(x_i))^{1-\beta(1-\alpha)}\right\}\right] \quad (3)$$

Taking (2) and (3) together, we get (ii).

(iv) $I(A,\overline{A})=I(\overline{A},A)$

Proof: L.H.S.:

$$I(A,\overline{A})=-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_{\overline{A}}(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{A}}(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_A(x_i)^{\beta(1-\alpha)}(1-\mu_A(x_i)^{1-\beta(1-\alpha)})+(1-\mu_A(x_i))^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}\right\}\right]$$

(4)

R.H.S.:

$$I(\overline{A},A)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{\overline{A}}(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{\overline{A}}(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{(1-\mu_A(x_i)^{\beta(1-\alpha)})\mu_A(x_i)^{1-\beta(1-\alpha)}+\mu_A(x_i)^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

(5)

Comparing (4) and (5), we get L.H.S. = R.H.S.

(v) $I(\overline{A}, \overline{B}) = I(A, B)$

Proof: We have

$$I(\overline{A}, \overline{B}) = -\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{\overline{A}}(x_i)^{\beta(1-\alpha)}\mu_{\overline{B}}(x_i)^{1-\beta(1-\alpha)} + (1-\mu_{\overline{A}}(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{B}}(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$= -\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)} + \mu_A(x_i)^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}\right\}\right]$$

$$= I(A, B).$$

(vi) $I(A, \overline{B}) = I(\overline{A}, B)$.

Proof: We have

$$I(A, \overline{B}) = -\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_{\overline{B}}(x_i)^{1-\beta(1-\alpha)} + (1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{B}}(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$= -\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_A(x_i)^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)} + (1-\mu_A(x_i))^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}\right\}\right]$$

(6)

$$I(\overline{A}, B) = -\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\mu_{\overline{A}}(x_i)^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)} + (1-\mu_{\overline{A}}(x_i))^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

$$= -\frac{\beta}{1-\alpha}\log_D\left[\frac{1}{n}\sum_{i=1}^{n}\left\{(1-\mu_A(x_i))^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)} + \mu_A(x_i)^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)}\right\}\right]$$

(7)

Comparing (6) and (7), we get $I(A, \overline{B}) = I(\overline{A}, B)$.

(vii) $I(A, B) + I(\overline{A}, B) = I(\overline{A}, \overline{B}) + I(A, \overline{B})$.

Proof: It is obvious from (v) and (vi) that (vii) holds.

IV. CONCLUSION:

In this paper, we have defined a two parametric fuzzy divergence measure and have also validated it. We also proved some of the properties of the given fuzzy divergence measure. Further, with the help of an example, the properties are satisfied.

REFERENCES

[1] Bhandari D. and Pal N.R. Some new information measures for fuzzy sets. *Information sciences 67*, 1992, 209-228.

[2] Kullback S. and Leibler R.A.. On information and sufficiency. *Annals of mathematical statistics 22*, 1951, 79-86.

[3] Kapur J.N. *measures of fuzzy information*. Mathematical sciences trust society, New Delhi, 1997.

[4] Havrda J.H. and Charvat F. Quantification methods of classification processes: concepts of structural alpha entropy, *Kybernetika 3*, 1967, 30-35.

[5] Hooda D.S. and Bajaj R.K. 'Useful' fuzzy measures of information, integrated ambiguity and directed divergence. *International Journal of General systems 39(6)*, 2010, 647-658.

[6] Verma R. and Sharma B.D. On generalized exponential fuzzy entropy. *World Academy of Science, Engineering and Technology 60*, 2011, 1402-1405.

[7]Tomar V.P. and Anshu Ohlan. New Parametric generalized exponential fuzzy divergence measure. *Journal of uncertainty analysis and applications 2*, 2014, 1-14..

[8] Saima Manzoor Sofi, S. Peerzada and M.A.K. Baig 'Useful' Fuzzy Divergence Measure and its Properties (*communicated*).

[9] S. Peerzada, Saima Manzoor Sofi and M.A.K. Baig. A New Two Parametric Fuzzy Information Measure and Its Properties. *International Journal of Advance Research in Science and Engineering 12(6)*, 2017, 1647-1654.