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Some Characterization Results of Lifetime Distributions using Two Parametric Shift-Dependent Generalized Dynamic (Residual) Information Measure

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ABSTRACT

In this paper, we introduce the concept of a shift-dependent generalized information measure of order α *and type* β *and its dynamic (residual) version. These are "length-biased" shift-dependent information measures that assign the larger weight to the larger values of the observed random variable. We derive the expressions of these two measures under the consideration of some well-known lifetime distributions. It is shown that the weighted generalized residual entropy determines the survival function uniquely. Some important properties and inequalities of the proposed residual information measure have also been discussed.*

Keywords: Shannon's entropy, generalized entropy, Shift-dependent dynamic information measure, Characterization results.

AMS subject Classification: 94A17, 94A24

1. INTRODUCTION

A very important measure of uncertainty in the the literature of information theory was originally introduced by Shannon [1]. For a non-negative absolutely continuous lifetime random variable X having probability density function $f(x)$, the measure is defined as

$$H(X)=-\int_0^\infty f(x)\log f(x)dx=-E[\log f(x)]. \tag{1}$$

and for a discrete random variable X taking values $x_i,\ i=1,2,...,n$ with respective probabilities $P(X=i)=p_i, 0\le p_i\le 1, \sum_{i=1}^{n} p_i=1$, it is defined as

$$H(P)=H(p_1,p_2,...,p_n)=-\sum_{x=1}^{n} p_x\log p_x. \tag{2}$$

Shannon's measure of entropy considers the outcomes of the random variable X equally important with respect to the goal set by the experimenter and hence they are given the same weight. But, in real life sometimes the elementary events of a probabilistic experiment have different qualitative characteristic which are usually known as the weight. So, in order to measure the entropy in such type of cases, Shannon's measure [1] is not useful. Belis and Guiasu [2] have proposed a new measure, known as weighted (useful) entropy which measures the uncertainty in such type of problems and is defined as

$$H^{w}(X) = -\int_{0}^{\infty} x f(x) \log f(x) dx = -E[X \log f(x)] . \tag{3}$$

where the importance of the occurrence of the event $X = x$ is represented by the coefficient x in the integral of (3) and is known as weight (see Misagh and Yari [3]). This is a length-biased shift-dependent information measure which ascribes the larger weight to the larger values of the observed random variable X . The notion of weighted residual entropy was given by Di Cescenzo and Longobardi [4] and is defined as

$$H^{w}(X;t) = -\int_{t}^{\infty} x \frac{f(x)}{\bar{F}(t)} \log \frac{f(x)}{\bar{F}(t)} dx . \tag{4}$$

Futher , weighted generalized information measure was studied by many researchers [5,6,7,8,9]. Sekeh, Borzadaran and Roknabadi [10] obtained some results based on weighted dynamic entropies. Nair, Sathar and Rajesh [11] studied dynamic weighted failure entropy. M. Nourbakhsh and G. Yari [12] introduced the concept of Weighted Renyi's entropy for lifetime distributions.

In the field of information theory, various authors have proposed different generalizations of Shannon's entropy [1]. Consequently, in this paper we consider a new generalized information measure consisting on two parameters α and β . Let X be an absolutely continuous non-negative random variable with probability density function (p.d.f) $f(x)$, then the generalized entropy (GE) is defined as

$$H_{\alpha,\beta}(X) = \frac{\alpha}{\beta(\beta-\alpha)} \log \int_{0}^{\infty} f^{\alpha-\beta+1}(x) dx, \; 0 < \alpha < \beta, \beta \geq 1, \tag{5}$$

where,

$$\lim_{\substack{\alpha \to 1 \\ \beta = 1}} H_{(\alpha,\beta)}(X) = -\int_{0}^{\infty} f(x) \log f(x) dx, \text{ which is Shannon's entropy given in (1).}$$

Considering this new and important generalized information measure, we study its weighted dynamic (residual) version which is defined as weighted generalized residual entropy of order α and type β . The rest of the paper is organized as follows: In section 2, we define weighted generalized entropy along with the expressions for some lifetime distributions. Section 3 expresses the weighted generalized residual information measure with some important expressions. We also focus on a characterization result which shows that the proposed measure determines the survival function uniquely. In section 4, the monotonic behavior of the measure with respect to exponential distribution is studied. Some important properties and inequalities of weighted generalized residual entropy are obtained in section 5. Finally in section 6, some concluding remarks are given.

2. Weighted Generalized Entropy (WGE)

Analogous to the definition of weighted entropy (3) , in this section, we study the weighted version of the generalized entropy (5).

Definition 2.1 For an absolutely continuous non-negative random variable X having density function $f(x)$, the weighted generalized entropy of order α and type β denoted by $H^w_{(\alpha,\beta)}(X)$ and is defined as

$$H^w_{(\alpha,\beta)}(X) = \frac{\alpha}{\beta(\beta-\alpha)}\log\left(\int_0^\infty (xf(x))^{\alpha-\beta+1}dx\right),\ \beta-1<\alpha<\beta,\ \beta\geq 1. \tag{6}$$

where the larger values of the random variable X are given the higher weight. In the following example, it is shown that even if the two distributions have the same GE, but they can have different WGE.

Example 2.1 Let the two non-negative random variables X and Y have the following density functions

$$f_X(x) = \begin{cases} \frac{1}{2},\ 4\leq x\leq 6 \\ 0,\ otherwise \end{cases}$$

and

$$f_Y(y) = \begin{cases} \frac{1}{2},\ 0\leq x\leq 2 \\ 0,\ otherwise. \end{cases}$$

For $\alpha=0.5$ and $\beta=1.4$, $H_{(\alpha,\beta)}(X)=H_{(\alpha,\beta)}(Y)=0.3887$, but $H^w_{(\alpha,\beta)}(X)=0.6071$ and $H^w_{(\alpha,\beta)}(Y)=0.3543$. Therefore, we observe that even though the generalized entropy of the random variable X is same as that of Y, but their weighted generalized entropies are not identical.

In table 1, we calculate the expressions of weighted generalized entropy corresponding to some well-known lifetime distributions are. Here, it is noted that $\Gamma(n,my)=m^n\int_y^\infty e^{-mz}z^{n-1}dz$ is an upper incomplete gamma function.

3. Weighted Generalized Residual Entropy

As the concept of residual entropy was introduced by Ebrahimi [13], but the concept of weighted residual entropy has been given by Di-Crescenzo and Longobardi [4] and is defined as

$$H^w(X;t) = -\int_t^\infty x\frac{f(x)}{\bar{F}(t)}\log\frac{f(x)}{\bar{F}(t)}dx. \tag{7}$$

On the basis of (7), the dynamic version of (6) is the weighted generalized entropy of the random variable $X_t=[X-t\,|\,X>t]$ and is defined as

$$H^w_{(\alpha,\beta)}(X;t) = \frac{\alpha}{\beta(\beta-\alpha)}\log\left(\int_t^\infty\left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}dx\right),\ \beta-1<\alpha<\beta,\ \beta\geq 1. \tag{8}$$

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when, $t=0$, (8) reduces to (6).

Table 1. Expressions of WGE $H^{w}_{(\alpha,\beta)}(X)$ for some lifetime distributions

Distribution	$f(x)$	x	$H^{w}_{(\alpha,\beta)}(X)$
Uniform	$\frac{1}{b-a}$	$a<x<b$	$p\log\left\{\frac{b^{r+1}-a^{r+1}}{(r+1)(b-a)^{r}}\right\}$
Exponential	$\lambda e^{-x\lambda}$	$x>0,\lambda>0$	$p\log\left\{\frac{\Gamma(r+1)}{\lambda r^{r+1}}\right\}$
Gamma	$\frac{1}{\Gamma(\lambda)}e^{-x}x^{\lambda-1}$	$0<x<\infty,\lambda>0$	$p\log\left\{\frac{\Gamma(\lambda r+1)}{(\Gamma(\lambda))^{r}r^{\lambda r+1}}\right\}$
Weibull	$\frac{1}{\theta}e^{-\left(\frac{x-\lambda}{\theta}\right)}$	$x>\lambda,\lambda>0,\theta>0$	$p\left[\frac{\lambda r}{\theta}+\log\left\{\frac{\theta\Gamma\left(r+1,\frac{\lambda}{\theta}r\right)}{r^{r+1}}\right\}\right]$
Pareto	$\frac{ab^{a}}{x^{a+1}}$	$x\geq b,b>0,a>0$	$p\log\left\{\frac{ba^{r}}{ar-1}\right\}$
Lomax	$\frac{\mu}{(1+x)^{\mu+1}}$	$x\geq 0,\mu>0$	$p\log\left\{\frac{\mu^{r}\Gamma(r+1)\Gamma(\mu r-1)}{\Gamma(r(1+\mu))}\right\}$

where, $p=\dfrac{\alpha}{\beta(\beta-\alpha)}$ and $r=\alpha-\beta+1$.

An alternative way of expressing (8) is obtained in the following theorem.

Theorem 3.1 For all $t>0$, we have the following equality

$$H^{w}_{(\alpha,\beta)}(X;t)=\frac{\alpha}{\beta(\beta-\alpha)}\log\left[t^{\alpha-\beta+1}\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H_{(\alpha,\beta)}(X;t)\right)\right.$$
$$\left.+(\alpha-\beta+1)\int_{t}^{\infty}z^{\alpha-\beta+1}\left(\frac{\bar{F}(z)}{\bar{F}(t)}\right)^{\alpha-\beta+1}\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}\right)H_{(\alpha,\beta)}(X;z)dz\right]\quad. \tag{9}$$

Proof.

$$\int_{t}^{\infty}\left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha+\beta-1}dx=\int_{t}^{\infty}\left(\int_{0}^{x}(\alpha-\beta+1)z^{\alpha-\beta+1}dz\right)\left(\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}dx$$

$$=(\alpha+\beta-1)\int_{t}^{\infty}\left[\int_{0}^{t}z^{\alpha-\beta+1}dz+\int_{t}^{x}z^{\alpha-\beta+1}dz\right]\left(\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}dx$$

$$=t^{\alpha-\beta+1}\int_t^{\infty}\left(\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}dx+(\alpha-\beta+1)\int_{z=t}^{\infty}\left(\frac{1}{\bar{F}(t)}z^{\alpha-\beta+1}\left(\int_{x=z}^{\infty}f^{\alpha-\beta+1}(x)dx\right)\right)dz\,.$$

Since,

$$\int_t^{\infty}\left(\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}dx=\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H_{(\alpha,\beta)}(X;t)\right).\qquad(10)$$

Therefore, due to (8) and (10), (9) is obtained.

In the following theorem, it is shown that $H^{w}_{(\alpha,\beta)}(X;t)$ characterizes the survival function $\bar{F}(t)$ uniquely.

Theorem 3.2 Let X be a non-negative random variable having probability density function $f(x)$ and survival function $\bar{F}(t)$. Assume that $H_{(\alpha,\beta)}(X;t)<\infty$, $\beta-1<\alpha<\beta$, $\beta\geq 1$ and increasing in t, then $H^{w}_{(\alpha,\beta)}(X;t)$ determines the survival function uniquely.

Proof. Rewriting (8) as

$$\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)=\int_t^{\infty}\left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}dx\,.\qquad(11)$$

Differentiating (11) w.r.t t, we have

$$\frac{d}{dx}\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)=(\alpha-\beta+1)\lambda_F(t)\int_t^{\infty}\left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}dx-(t\lambda_F(t))^{\alpha-\beta+1}.\qquad(12)$$

where $\lambda_F(t)=\frac{f(t)}{\bar{F}(t)}$ denotes the failure rate of X. Using (11), we can rewrite (12) as

$$(t\lambda_F(t))^{\alpha-\beta+1}-(\alpha-\beta+1)\lambda_F(t)\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)+\frac{d}{dx}\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)=0$$

Hence for fixed $t>0$, $\lambda_F(t)$ is a solution of $\psi(x)=0$, where

$$\psi(x)=t^{\alpha-\beta+1}x^{\alpha-\beta+1}-(\alpha-\beta+1)x\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)+\frac{d}{dx}\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)$$

Differentiating both sides w.r.t x, we have

$$\psi'(x)=(\alpha-\beta+1)t^{\alpha-\beta+1}x^{\alpha-\beta}-(\alpha-\beta+1)\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}\right).$$

Now, $\psi'(x)=0$ gives

$$x_t=\left[t^{\beta-\alpha-1}\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)\right]^{\alpha-\beta+1}.$$

Also,

$$\psi''(x)=(\alpha-\beta+1)(\alpha-\beta)t^{\alpha-\beta+1}x^{\alpha-\beta-1}.$$

For $\beta-1<\alpha<\beta$, $\beta\geq 1$, $\psi''(x_t)<0$. Therefore, $\psi(x)$ attains maximum at x_t. So, $x=\lambda_F(t)$ is the unique solution to $\psi(x)=0$. Thus, $H^{w}_{(\alpha,\beta)}(X;t)$ determines $\lambda_F(t)$ uniquely, which in turns determines $\bar{F}(t)$ uniquely.

In table 2, we derive the expressions of weighted generalized residual entropy corresponding to some well-known lifetime distributions. It is mentioned that $\Gamma(n,mz)=m^n\int_z^\infty y^{n-1}e^{-my}dy, m>0, n>0$ and $\gamma(n,mz)=m^n\int_0^z y^{n-1}e^{-my}dy, m>0, n>0$ are the upper and lower incomplete gamma functions respectively.

Table 2. Weighted generalized residual entropy $H^w_{(\alpha,\beta)}(X;t)$ of some lifetime distributions

Distribution	$f(x)$	x	$H^w_{(\alpha,\beta)}(X;t)$
Uniform	$\frac{1}{b-a}$	$a<x<b$	$p\log\left\{\frac{b^{r+1}-t^{r+1}}{(r+1)(b-t)^r}\right\}$
Exponential	$\theta e^{-x\theta}$	$x>0,\ \theta>0$	$p\left[rt\theta+\log\left\{\frac{\Gamma(r+1,\theta rt)}{\theta r^{r+1}}\right\}\right]$
Gamma	$\frac{1}{\Gamma(b)}e^{-x}x^{b-1}$	$0<x<\infty,\ b>0$	$p\log\left\{\frac{\Gamma(br+1,rt)}{r^{br+1}(\Gamma(b)-\gamma(b,t))^r}\right\}$
Pareto	$\frac{\theta\lambda^\theta}{x^{\theta+1}}$	$x\geq\lambda,\ \lambda>0,\ \theta>0$	$p\log\left\{\frac{t\theta^r}{\theta r-1}\right\}$
Weibull	$\frac{1}{b}e^{-\left(\frac{x-a}{b}\right)}$	$x>a, a>0, b>0$	$p\left[\frac{tr}{b}+\log\left\{\frac{b\Gamma\left(r+1,\frac{r}{b}t\right)}{r^{r+1}}\right\}\right]$

where, $p=\frac{\alpha}{\beta(\beta-\alpha)}$ and $r=\alpha-\beta+1$.

4. Monotonic behavior of $H^w_{(\alpha,\beta)}(X;t)$

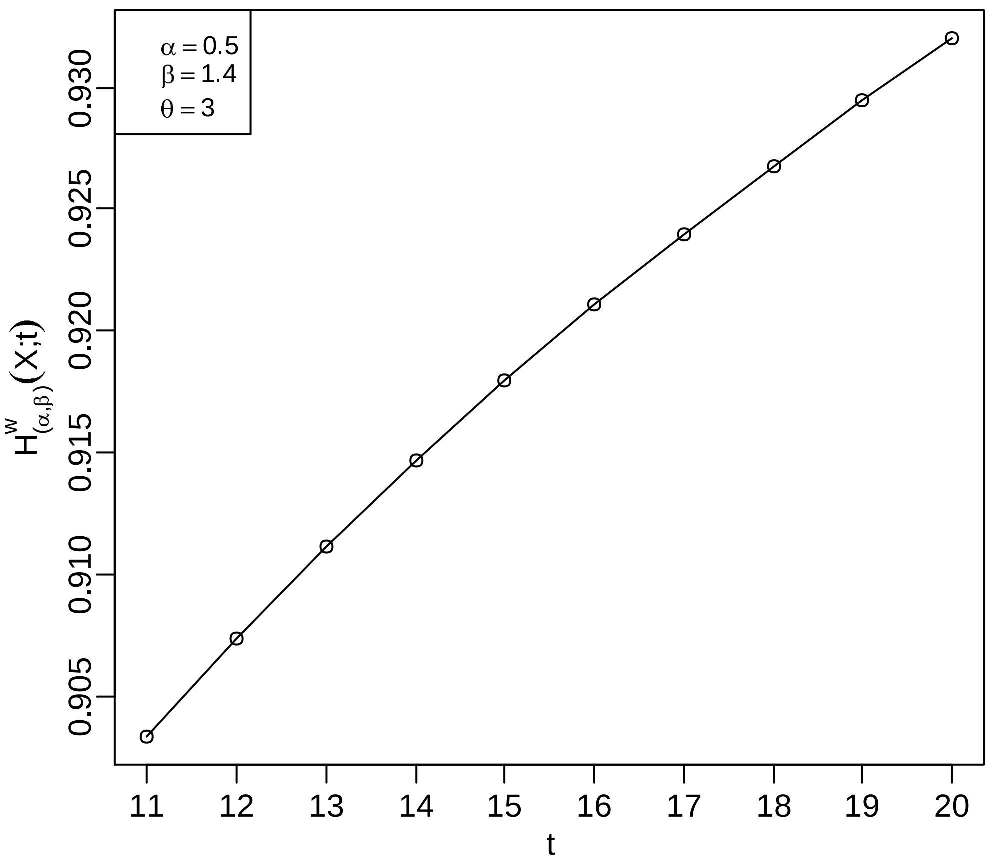
In this section, we study the monotonic behavior of the generalized residual entropy $H^w_{(\alpha,\beta)}(X;t)$ with respect to exponential distribution.

In table 2, assuming $\alpha=0.5$, $\beta=2.5$ and $\theta=3$ in the expression of $H^w_{(\alpha,\beta)}(X;t)$ corresponding to exponential distribution and then calculate the values of the expression for different values of t as shown in the following table.

t	11	12	13	14	15	16	17	18	19	20
$H^w_{(\alpha,\beta)}(X;t)$	0.9034	0.9074	0.9112	0.9147	0.9180	0.9211	0.9240	0.9268	0.9295	0.9320

The graph of this table is shown in Fig.1 and it is obvious that $H^{w}_{(\alpha,\beta)}(X;t)$ is monotonic increasing in $t \in [11,20]$.

Fig. 1. Weighted Generalized Residual Entropy for Exponential Distribution



5. Properties and inequalities of $H^{w}_{(\alpha,\beta)}(X;t)$

In this section, we study some important properties and inequalities of weighted generalized residual entropy $H^{w}_{(\alpha,\beta)}(X;t)$.

Definition 5.1 Let X and Y be two non-negative random variables, then X is said to be smaller than Y in weighted (useful) generalized residual entropy (denoted by $X \overset{WGRE}{\leq} Y$) if $H^{w}_{(\alpha,\beta)}(X;t) \leq H^{w}_{(\alpha,\beta)}(Y;t)$, for all $t > 0$.

Definition 5.2 A survival function $\bar{F}$ is said to have increasing (decreasing) weighted (useful) generalized residual entropy IWGRE (DWGRE) if $H^{w}_{((\alpha,\beta))}(X;t)$ is increasing (decreasing) in t, $t>0$, i.e if $H'^{w}_{(\alpha,.\beta)}(X;t)>(<)0$.

Example 5.1 Let X be an exponentially distributed random variable having pdf $f(x)=\theta e^{-x\theta}$, $x>0, \theta>0$, then from table 2, we have

$$H'^{w}_{(\alpha,\beta)}(X;t)=\frac{\alpha}{\beta(\beta-\alpha)}\left(r\theta+\frac{\Gamma'(r+1,\theta rt)}{\Gamma(r+1,\theta rt)}\right), \text{ where, } r=\alpha-\beta+1$$

if, $\beta>\alpha$, then we obtain $\bar{F}$ as IWGRE.

The following lemma will be very useful in proving the theorems of this section.

Lemma 5.1 For an absolutely continuous random variable X, define $Z=aX$, where $a>0$ is a constant, then

$$H^{w}_{(\alpha,\beta)}(Z;t)=\frac{\alpha}{\beta(\beta-\alpha)}\log a+H^{w}_{(\alpha,\beta)}(X;t). \tag{13}$$

proof. $H^{w}_{(\alpha,\beta)}(Z;t)=\dfrac{\alpha}{\beta(\beta-\alpha)}\log\displaystyle\int_{t}^{\infty}\left(\frac{z}{a}\cdot\frac{f\left(\frac{z}{a}\right)}{\Pr(Z>t)}\right)^{\alpha-\beta+1} dz$.

Setting $Z=aX$, a strictly increasing function of X, we have

$$H^{w}_{(\alpha,\beta)}(X;t)=\frac{\alpha}{\beta(\beta-\alpha)}\log\left[a\int_{\frac{t}{a}}^{\infty}\left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}dx\right].$$

Using (8), (13) is obtained.

Theorem 5.1 Let $\bar{F}$ be an IWGRE (DWGRE) and $\beta>\alpha$, then

$$\lambda_F(t)\le(\ge)\left[\frac{(\alpha-\beta+1)\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)}{t^{\alpha-\beta+1}}\right]^{\frac{1}{\alpha-\beta}}.$$

Proof . From (8), we have

$$\frac{\beta(\beta-\alpha)}{\alpha}H'^{w}_{(\alpha,\beta)}(X;t)=\lambda_F(t)\left[(\alpha-\beta+1)-t\left(t\lambda_F(t)\right)^{\alpha-\beta}\exp\left(-\frac{\beta(\beta-\alpha)}{\alpha}H^{w}_{(\alpha,\beta)}(X;t)\right)\right].$$

Since, $\bar{F}$ is IWGRE (DWGRE) and $\beta>\alpha$, then we obtain

$$\lambda_F(t)\left[(\alpha-\beta+1)-t\left(t\lambda_F(t)\right)^{\alpha-\beta}\exp\left(-\frac{\beta(\beta-\alpha)}{\alpha}H^w_{(\alpha,\beta)}(X;t)\right)\right]\geq(\leq)0 .$$

which gives

$$\lambda_F(t)\leq(\geq)\left[\frac{(\alpha-\beta+1)\exp\left(\frac{\beta(\beta-\alpha)}{\alpha}H^w_{(\alpha,\beta)}(X;t)\right)}{t^{\alpha-\beta+1}}\right]^{\frac{1}{\alpha-\beta}} .$$

hence the desired result is obtained.

Theorem 5.2 Let X and Y be two absolutely continuous non-negative random variables, define $Z_1=a_1X$ and $Z_2=a_2X$, $a_1,a_2>0$ are constants. Let $X\overset{WGRE}{\leq}Y$ and $a_1\leq a_2$. Then, $Z_1\overset{WGRE}{\leq}Z_2$, if $H^w_{(\alpha,\beta)}(X;t)$ or $H^w_{(\alpha,\beta)}(Y;t)$ is decreasing in $t>0$.

Proof. Suppose $H^w_{(\alpha,\beta)}(X;t)$ is decreasing in t.

Now, $X\overset{WGRE}{\leq}Y$ implies

$$H^w_{(\alpha,\beta)}\left(X;\frac{t}{a_2}\right)\leq H^w_{(\alpha,\beta)}\left(Y;\frac{t}{a_2}\right). \tag{14}$$

Also, $\frac{t}{a_1}\geq\frac{t}{a_2}$ gives

$$H^w_{(\alpha,\beta)}\left(X;\frac{t}{a_1}\right)\leq H^w_{(\alpha,\beta)}\left(X;\frac{t}{a_2}\right) . \tag{15}$$

From (14) and (15), we get

$$H^w_{(\alpha,\beta)}\left(X;\frac{t}{a_1}\right)\leq H^w_{(\alpha,\beta)}\left(Y;\frac{t}{a_2}\right) . \tag{16}$$

Using (35) and applying lemma 4.1, we get $Z_1\overset{WGRE}{\leq}Z_2$.

Theorem 5.3 Let X be an absolutely continuous non-negative random variable and $X\in IWGRE(DWGRE)$. Define $Z\in aX$, where $a>0$ is a constant. Then $Z\in IWGRE(DWGRE)$.

Proof. Since $X\in IWGRE(DWGRE)$,

Therefore,

$$H'^{w}_{(\alpha,\beta)}(X;t) \geq 0$$
$$\leq 0.$$

By applying lemma 4.1, it is obvious that $Z \in IWGRE(DWGRE)$ and hence the theorem is proved.

Here, we study some inequalities on the basis of $H^{w}_{(\alpha,\beta)}(X;t)$.

Theorem 5.4 Let X be the lifetime of a system with p.d.f $f(x)$ and survival function $\bar{F}(t), t>0$, then for $\beta > 0$, the following inequality is obtained

$$H^{w}_{(\alpha,\beta)}(X;t) \geq \frac{\alpha(\alpha-\beta+1)}{\beta(\beta-\alpha)} \int_t^\infty \frac{f(x)\log x}{\bar{F}(t)} dx - \frac{\alpha}{\beta} H(X;t).$$

Proof. we know that from log-sum inequality

$$\int_t^\infty f(x) \log \frac{f(x)}{\left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1}} dx \geq \int_t^\infty f(x)dx \log \frac{\int_t^\infty f(x)dx}{\int_t^\infty \left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1} dx} . \tag{17}$$

where (17) is obtained from (8) .

The L.H.S of (17) leads to

$$(\beta-\alpha)\int_t^\infty f(x)\log f(x)dx - (\alpha-\beta+1)\int_t^\infty f(x)\log x dx + (\alpha-\beta+1)\bar{F}(t)\log \bar{F}(t). \tag{18}$$

Substituting (18) in (17) and after simplification we get the desired result.

Theorem 5.6 Let X be a non-negative random variable with support $(0,b]$ and having probability density function p.d.f $f(x)$, survival function $\bar{F}(t), t>0$, then for $\beta > \alpha$, the following inequality holds.

$$H^{w}_{(\alpha,\beta)}(X;t) \leq \frac{\alpha}{\beta(\beta-\alpha)} \left[\frac{\int_t^b \left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1} \log\left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1} dx}{\int_t^b \left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1} dx} + \log(b-t) \right] .$$

Proof. From log-sum inequality and (8), we have

$$\int_t^b \left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1} \log\left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1} dx \geq \int_t^b \left(x\frac{f(x)}{\bar{F}(t)}\right)^{\alpha-\beta+1} dx \log \frac{\int_t^b (xf(x))^{\alpha-\beta+1}}{\int_t^b (\bar{F}(t))^{\alpha-\beta+1} dx}$$

$$=\int_t^b\left(x\frac{f(x)}{\overline{F}(t)}\right)^{\alpha-\beta+1}dx\left[\frac{\beta(\beta-\alpha)}{\alpha}H^w_{(\alpha,\beta)}(X;t)-\log(b-t)\right].$$

After simplification, we get the desired result.

Preposition 5.1 Let X be a non-negative random variable with WGRE $H^w_{(\alpha,\beta)}(X;t)$, then for $\beta>\alpha$, the following inequality holds.

$$H^w_{(\alpha,\beta)}(X;t)\le\frac{\alpha}{\beta(\beta-\alpha)}\left(\int_t^\infty\left(x\frac{f(x)}{\overline{F}(t)}\right)^{\alpha-\beta+1}dx-1\right).$$

Proof. We know that $\log x\le x-1$, therefore, from (8), we obtain

$$H^w_{(\alpha,\beta)}(X;t)\le\frac{\alpha}{\beta(\beta-\alpha)}\left(\int_t^\infty\left(x\frac{f(x)}{\overline{F}(t)}\right)^{\alpha-\beta+1}dx-1\right).$$

6. Conclusion

In this paper, we have introduced and studied the concept of weighted generalized entropy and its dynamic (residual) version. We derive the expressions of these measures corresponding to some well-known lifetime distributions. It is shown that the weighted generalized residual entropy characterizes the distribution function uniquely. Further, we study the monotonic behavior of the proposed dynamic measure on the basis of exponential distribution. Finally various properties and inequalities of the measure have also been studied.

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