IJARSE
ISSN: 2319-8354

Common fixed point theorem for six mappings with compatibility of type (β) on intuitionistic fuzzy metric space

[1]Shamim Ahmad Thoker, [2]Bhavana Deshpande, [3]Amrish Handa
Department of Mathematics, Govt. P. G. Arts & Science College,
Ratlam (M. P.) India

Abstract. The purpose of this paper is to prove some common fixed point theorem for six mappings satisfying condition of compatibility of type (β) on complete intuitionistic fuzzy metric space by using different type of sequence. Our result extend, generalize and intuitionistically fuzzify several known results in fuzzy metric spaces.

AMS (2000) subject classification: 47H10, 54H25.

Keywords: Intuitionistic fuzzy metric space, compatible maps of type (β), common fixed point.

Introduction. Sessa [18] generalized notion of commutativity and defined weak commutativity. Jungck [8] introduced more generalized commutativity so called compatibility.

Zadeh [23] introduced the concept of fuzzy set as a new way to represent vagueness in our everyday life. A fuzzy set A in X is a function whose domain is X and which gives the values in the interval $[0, 1]$. When the uncertainty occur due to the fuzziness rather than randomness as in the measurement of an ordinary length then the concept of fuzzy metric space is more suitable. The concept of fuzzy metric space has become the area of interest for researcher due to its vast applicability such as to achieve access optimization in information system, history prediction, image filtering and product spaces.

George and Veeramani [5] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [12] and defined the Hausdorff topology of fuzzy metric spaces and showed that every metric space induces a fuzzy metric space.

Mishra, Sharma and Singh [15] introduced the concept of compatibility in fuzzy metric spaces. Further compatible maps of type (α) introduced by Cho [2] and compatible maps of type (β) introduced by Cho et al. [3]. Grabiec [6], Kramosil and Michalek [12], Mishra et al. [15] obtained the common fixed point theorem for compatible maps on fuzzy metric spaces.

Alaca et al. [1] generalized fuzzy metric space due to Kramosil and Michalek [12] and defined the notion of intuitionistic fuzzy metric space as Park [16] with the help of continuous t-norms and continuous t-conorms. Turkoglu et al. [21] introduced the concept of compatible maps of type (α) and type (β) in intuitionistic fuzzy metric spaces.

Many authors studied fixed point theorems on intuitionistic fuzzy metric space including Gregory et al. [7], Saadati and Park [17] and Turkoglu et al. [21]. Sharma and Deshpande [20] improved the result of Cho, Pathak, Kang and Jung [3] and also generalized and fuzzified several fixed point theorems on metric spaces.

In this paper, we prove a common fixed point theorem for six mappings under the compatibility of type (β) on complete intuitionistic fuzzy metric space by using a different type of sequence. Our result intuitionistically fuzzifies the result of Sharma and Deshpande [20] and also generalizes several known results. We also give an example to validate our main theorem.

Preliminaries

Definition 1. [**19**]. A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if it satisfies the following conditions:

(1) $*$ is commutative and associative.

(2) $*$ is continuous.

(3) $a * 1 = a$ for all $a \in [0, 1]$.

(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2. [**19**]. A binary operation $\diamondsuit : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if it satisfies the following conditions:

(1) $\diamondsuit$ is commutative and associative.

(2) $\diamondsuit$ is continuous.

(3) $a \diamondsuit 0 = a$ for all $a \in [0, 1]$.

(4) $a \diamondsuit b \leq c \diamondsuit d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

IJARSE
ISSN: 2319-8354

Remark 1. The concept of triangular norms (t- norm) and triangular conorms (t-conorm) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions respectively. These concepts were originally introduced by Menger [14] in his study of statistical metric spaces. Several examples for these concepts were purposed by many authors [4, 10, 11, 22].

Definition 3. [1] A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M and N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

(1) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$.
(2) $M(x, y, 0) = 0$ for all $x, y \in X$.
(3) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ iff $x = y$.
(4) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$.
(5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s)$ for all $x, y, z \in X$ and $s, t > 0$.
(6) for all $x, y \in X$, $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous.
(7) $lim_{t \to \infty} M(x, y, t) = 1$ for all $x, y \in X$.
(8) $N(x, y, 0) = 1$ for all $x, y \in X$.

(9) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$.
(10) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > o$.
(11) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t+s)$ for all $x, y, z \in X$ and $t, s > 0$.
(12) for all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ is right continuous.
(13) $lim_{t \to \infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The function $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non nearness between x and y with respect to t respectively.

Remark 2. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, N, *, \Diamond)$ such that t-norm $*$ and t-conorm $\Diamond$ are associated [13] i.e. $x \Diamond y = 1 - (1-x) * (1-y)$ for all $x, y \in X$.

Example 1. Let (X, d) be a metric space. Define t-norm $a * b = \min\{a, b\}$ and t-conorm $a \Diamond b = \max\{a, b\}$ and for all $x, y \in X$ and $t > 0$

$M_d(x, y, t) = \frac{t}{t+d(x,\, y)}$, $N_d(x, y, t) = \frac{d(x,\, y)}{t+d(x,\, y)}$

Then $(X,\, M,\, N,\, *,\, \Diamond)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric $(M,\, N)$ introduced by the metric d the standard intuitionistic fuzzy metric.

Remark 3. In intuitionistic fuzzy metric space $(X,\, M,\, N,\, *,\, \Diamond)$, $M(x,\, y,\, \cdot)$ is non decreasing and $N(x,\, y,\, \cdot)$ is non increasing for all $x,\, y \in X$.

Definition 4. [**1**] Let $(X,\, M,\, N,\, *,\, \Diamond)$ be an intuitionistic fuzzy metric space. Then

(1) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if for all $t > 0$ and $p > 0$

$\lim_{n\to\infty} M(x_{n+p},\, x_n,\, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p},\, x_n,\, t) = 0$.

(2) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if for all $t > 0$

$\lim_{n\to\infty} M(x_n,\, x,\, t) = 1$ and $\lim_{n\to\infty} N(x_n, x,\, t) = 0$.

$*$ and $\Diamond$ are continuous, the limit is uniquely determined from (5) and (11) respectively.

Definition 5. [**1**] An intuitionistic fuzzy metric space $(X,\, M,\, N,\, *,\, \Diamond)$ is said to be complete iff every Cauchy sequence in X is convergent.

Lemma 1. Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X if there exist a number $k \in (0,\, 1)$ such that

(1) $M(y_{n+2},\, y_{n+1},\, kt) \geq M(y_{n+1},\, y_n,\, t)$

(2) $N(y_{n+2},\, y_{n+1},\, kt) \leq N(y_{n+1},\, y_n,\, t)$

for all $t > 0$ and $n = 1,\, 2, \ldots\ldots\ldots$ then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2. Let $(X,\, M,\, N,\, *,\, \Diamond)$ be an intuitionistic fuzzy metric space and for all $x,\, y \in X$ and $t > 0$ and for a number $k \in (0,\, 1)$

IJARSE
ISSN: 2319-8354

$M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ then $x = y$.

Definition 6. [**21**]. Let A and B be maps from intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ into itself. The maps A and B are said to be compatible if for all $t > 0$

$lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ and $lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$,
whenever $\{x_n\}$ is a sequence in X such that
$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition 7. [**21**]. Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ into itself. The maps A and B are said to be compatible maps of type (α) if for all $t > 0$

$\lim_{n\to\infty} M(ABx_n, BBx_n, t) = 1$, $\lim_{n\to\infty} N(ABx_n, BBx_n, t) = 0$,
and
$\lim_{n\to\infty} M(BAx_n, AAx_n, t) = 1$, $\lim_{n\to\infty} N(BAx_n, AAx_n, t) = 0$,
whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition 8. [**21**] Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ into itself. The maps A and B are said to be compatible maps of type (β) if for all $t > 0$

$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1$ and $\lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0$,
whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Remark 4. Turkoglu et al. [21] introduced the concept of compatible maps and compatible maps of type (α) and (β) in intuitionistic fuzzy metric space. In our theorems and corollaries $(X, M, N, *, \Diamond)$ will denote an intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\Diamond$ respectively defined by $t * t \geq t$ and $(1-t) \Diamond (1-t) \leq (1-t)$ for all $t \in [0, 1]$.

Main result

Theorem 1 Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric space. Let A, B, S, T, P and Q be six mappings from X into itself such that

(1.1) $P(ST)(X) \cup Q(AB)(X) \subset AB(ST)(X)$.

(1.2) there exist a constant $k \in (0, 1)$ such that

$$M(Px, Qy, kt) \geq M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t)$$
$$*M(Px, STy, \alpha t) * M(Qy, ABx, (2-\alpha)t)$$

and

$$N(Px, Qy, kt) \leq N(ABx, STy, t) \Diamond N(Px, ABx, t) \Diamond N(Qy, STy, t)$$
$$\Diamond N(Px, STy, \alpha t) \Diamond N(Qy, ABx, (2-\alpha)t)$$

for all $x, y \in X, \alpha \in (0, 2)$ and $t > 0$.

(1.3) $AB = BA$, $PB = BP$, $TQ = QT$, $ST = TS$, $(AB)(ST) = (ST)(AB)$.

(1.4) A, B, S and T are continuous.

(1.5) the pairs $\{P, AB\}$ and $\{Q, ST\}$ are compatible of type (β).

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof Let x_0 be an arbitrary point of X. Then by (1.1), we can construct a sequence $\{x_n\}$ in X as follows

$$\left.\begin{array}{l} y_{2n} = P(ST)x_{2n} = AB(ST)x_{2n+1} \\ y_{2n+1} = Q(AB)x_{2n+1} = AB(ST)x_{2n+2} \end{array}\right\} n = 0, 1, 2, \ldots\ldots$$

Put $x = STx_{2n+1}$, $y = ABx_{2n+2}$ in (1.2) with $\alpha = 1 - q$, $q \in (0, 1)$ and $t > 0$, we get

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)*M(y_{2n+1}, y_{2n}, t)*M(y_{2n+2}, y_{2n+1}, t)$$
$$*M(y_{2n+1}, y_{2n+1}, (1-q)t) * M(y_{2n+2}, y_{2n}, (1+q)t)$$

and

$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n}, t) \Diamond N(y_{2n+2}, y_{2n+1}, t)$$
$$\Diamond N(y_{2n+1}, y_{2n+1}, (1-q)t) \Diamond N(y_{2n+2}, y_{2n}(1+q)t).$$

$$\Longrightarrow M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)*M(y_{2n}, y_{2n+1}, t)*M(y_{2n+1}, y_{2n+2}, t)$$
$$*1 * M(y_{2n+2}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, qt)$$

and

$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}y_{2n+2}, t)$$
$$\Diamond 0 \Diamond N(y_{2n+2}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n}, qt).$$

$$\Longrightarrow M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)*M(y_{2n+1}, y_{2n+2}, t)*M(y_{2n}, y_{2n+1}, qt) \_\_\_\_ 1(a)$$

and

$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n+2}, t) \Diamond N(y_{2n}, y_{2n+1}, qt). \_\_\_\_ 1(b)$$

Since t-norm $*$ and t-conorm $\Diamond$ are continuous, $M(x, y, \cdot)$ is left continuous and $N(x, y, \cdot)$ is right continuous. Letting $q \to 1$ in $1(a)$ and $1(b)$, we get

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t)$$

and

$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n+2}, t).$$

Similarly

$$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n+2}, y_{2n+3}, t)$$

and

International Journal of Advance Research in Science and Engineering
Volume No.07, Special Issue No.04, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

$N(y_{2n+2}, y_{2n+3}, kt) \leq N(y_{2n+1}, y_{2n+2}, t) \Diamond N(y_{2n+2}, y_{2n+3}, t)$.

In general, we have for $m = 1, 2, \ldots\ldots$

$M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t) * M(y_{m+1}, y_{m+2}, t)$

and

$N(y_{m+1}, y_{m+2}, kt) \leq N(y_m, y_{m+1}, t) \Diamond N(y_{m+1}, y_{m+2}, t)$.

Consequently, it follows that $m, p = 1, 2, \ldots\ldots$

$M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t) * M(y_{m+1}, y_{m+2}, \frac{t}{k^p})$

and

$N(y_{m+1}, y_{m+2}, kt) \leq N(y_m, y_{m+1}, t) \Diamond N(y_{m+1}, y_{m+2}, \frac{t}{k^p})$.

$\because M(y_{m+1}, y_{m+2}, \frac{t}{k^p}) \to 1$ and $N(y_{m+1}, y_{m+2}, \frac{t}{k^p}) \to 0$ as $p \to \infty$.

So, we have for $m = 1, 2, \ldots\ldots$

$M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t)$

and

$N(y_{m+1}, y_{m+2}, kt) \leq N(y_m, y_{m+1}, t)$.

$\Longrightarrow \{y_n\}$ is a Cauchy sequence in X.

$\because (X, M, N, *, \Diamond)$ is complete and so $\{y_n\}$ converges to a point $z \in X$. Since $\{P(ST)x_{2n}\}$, $\{Q(AB)x_{2n+1}\}$ are subsequences of $\{y_n\}$ and so

$P(ST)x_{2n} \to z$ and $Q(AB)x_{2n+1} \to z$ as $n \to \infty$.

Let

$u_n = STx_n$, $v_n = ABx_n$ for $n = 1, 2, \ldots\ldots$

Then we have

$Pu_{2n} \to z$, $ABu_{2n} \to z$, $STv_{2n+1} \to z$, $Qv_{2n+1} \to z$ as $n \to \infty$.

$\because$ the pairs $\{P, AB\}$ and $\{Q, ST\}$ are compatible of type (β)

$\therefore M(PPu_{2n}, ABABu_{2n}, t) \to 1$, $N(PPu_{2n}, ABABu_{2n}, t) \to 0$

and

$M(QQv_{2n+1}, STSTv_{2n+1}, t) \to 1$, $N(QQv_{2n+1}, STSTv_{2n+1}, t) \to 0$ as $n \to \infty$. (2)

International Journal of Advance Research in Science and Engineering
Volume No.07, Special Issue No.04, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

Moreover by the continuity of A, B, S and T and by (2), we have
$ABPu_{2n} \to ABz$ and $PPu_{2n} \to ABz$,
$STQv_{2n+1} \to STz$ and $QQv_{2n+1} \to STz$ as $n \to \infty$.
Put $x = u_{2n}$, $y = Qv_{2n+1}$ in (1.2) with $\alpha = 1$ and taking limit as $n \to \infty$, we get
$M(z, STz, kt) \geq M(z, STz, t)$ and $N(z, STz, kt) \leq N(z, STz, t)$.
$\Longrightarrow STz = z.$ ___$3(a)$
Put $x = Pu_n$, $y = v_{2n+1}$ in (1.2) with $\alpha = 1$ and taking limit as $n \to \infty$, we get
$M(ABz, z, kt) \geq M(ABz, z, t)$ and $N(ABz, z, kt) \leq N(ABz, z, t)$.
$\Longrightarrow ABz = z.$
___$3(b)$
Put $x = u_{2n}$, $y = z$ in (1.2) with $\alpha = 1$ and taking limit as $n \to \infty$, we get
$M(z, Qz, kt) \geq M(z, Qz, t)$ and $N(z, Qz, kt) \leq N(z, Qz, t)$.
$\Longrightarrow Qz = z.$
___$3(c)$
Put $x = z$, $y = v_{2n+1}$ in (1.2) with $\alpha = 1$ and taking limit as $n \to \infty$, we get
$M(Pz, z, kt) \geq M(Pz, z, t)$ and $N(Pz, z, kt) \leq N(Pz, z, t)$.
$\Longrightarrow Pz = z.$ ___$3(d)$
Put $x = z$, $y = Tz$ in (1.2) with $\alpha = 1$, we get
$M(z, Tz, kt) \geq M(z, Tz, t)$ and $N(z, Tz, kt) \leq N(z, Tz, t)$
$\Longrightarrow Tz = z.$ ___$3(e)$
Put $x = Bz$, $y = z$ in (1.2) with $\alpha = 1$, we get
$M(Bz, z, kt) \geq M(Bz, z, t)$ and $N(Bz, z, kt) \leq N(Bz, z, t)$
$\Longrightarrow Bz = z.$ ___$3(f)$
By $3(a)$ and $3(e)$, we have
$Sz = z.$ ___$3(g)$
By $3(b)$ and $3(f)$, we have
$Az = z.$ ___$3(h)$
Thus by $3(c)$, $3(d)$, $3(e)$, $3(f)$, $3(g)$ and $3(h)$, we have
$Az = Bz = Sz = Tz = Pz = Qz = z.$
$\Longrightarrow A, B, S, T, P$ *and* Q have a common fixed point.

Uniqueness of fixed point

Let z and z_1 be two common fixed point of A, B, S, T, P *and* Q. Then
$Az = Bz = Sz = Tz = Pz = Qz = z$
and
$Az_1 = Bz_1 = Sz_1 = Tz_1 = Pz_1 = Qz_1 = z_1.$
z and $y = z_1$ in (1.2) with $\alpha = 1$, we get
$M(z, z_1, kt) \geq M(z, z_1, t)$ and $N(z, z_1, kt) \leq N(z, z_1, t)$.
$\Longrightarrow z = z_1.$
$\Longrightarrow z$ is unique.
Then A, B, S, T, P *and* Q have a common fixed point.
If we put $B = T = I$ in theorem, we have the following:

Corollary 1. Let $(X, M, N, *, \Diamond)$ be a complete IFM space and A, S, P *and* Q be mappings from X into itself such that

(1.1) $PS(X) \cup QA(X) \subset AS(X)$.

(1.2) there exist a constant $k \in (0, 1)$ such that

$$M(Px, Qy, kt) \geq M(Ax, Sy, t) * M(Px, Ax, t) * M(Qy, Sy, t) * M(Px, Sy, \alpha t) * M(Qy, Ax, (2-\alpha)t)$$

and

$$N(Px, Qy, kt) \leq N(Ax, Sy, t) \Diamond N(Px, Ax, t) \Diamond N(Qy, Sy, t) \Diamond N(Px, Sy, \alpha t) \Diamond N(Qy, Ax, (2-\alpha)t)$$

for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$.

(1.3) $AS = SA$.

(1.4) A and S are continuous.

(1.5) the pairs $\{P, A\}$and $\{Q, S\}$ are compatible maps of type (β).

Then A, S, P *and* Q have a unique common fixed point in X.

If we put $A = S$, $B = T$ and $P = Q$ in the theorem, we have the following:

Corollary 2. Let $(X, M, N, *, \Diamond)$ be a complete IFM space and A, B *and* P be mappings from X into itself such that

(1.1) $P(X) \subset AB(X)$.

(1.2) there exist a constant $k \in (0, 1)$ such that

$$M(Px, Py, kt) \geq M(ABx, ABy, t) * M(Px, ABx, t) * M(Py, ABy, t) * M(Px, ABy, \alpha t) * M(Py, ABx, (2-\alpha)t)$$

and

$$N(Px, Py, kt) \leq N(ABx, ABy, t) \Diamond N(Px, ABx, t) \Diamond N(Py, ABy, t) \Diamond N(Px, ABy, \alpha t) \Diamond N(Py, ABx, (2-\alpha)t)$$

for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$.

(1.3) $AB = BA$, $BP = PB$.

(1.4) A and B are continuous.

(1.5) the pair $\{P, AB\}$ are compatible of type (β).

Then A, B *and* P have a unique common fixed point in X.

Example 2. Let $X = [0, 1]$ with the metric d defined by

$d(x, y) = |\, x - y|$ for all $x, y \in X$.

Define

$M(x, y, t) = \frac{t}{t+|x-y|}$, $N(x, y, t) = \frac{|x-y|}{t+|x-y|}$ for all $x, y \in X$ and $t > 0$.
$M(x, y, 0) = 0$ and $N(x, y, 0) = 1$.

Clearly $(X, M, N, *, \diamondsuit)$ is a complete intuitionistic fuzzy metric space where $*$ and $\diamondsuit$ are defined by

$a * b = ab$ and $a \diamondsuit b = min\ \{1, a + b\}$.

Let A, B, S, T, P and Q be defined as

$Ax = x,\ Bx = \frac{x}{2},\ Sx = \frac{x}{3}$,

$$Tx = \begin{cases} x, & x \neq 1 \\ 1, & x = 1 \end{cases},$$

$$Px = \begin{cases} \frac{x}{4}, & x \leq \frac{1}{2} \\ \frac{1}{2} - \frac{x}{4}, & x > \frac{1}{2} \end{cases},$$

$$Qx = \begin{cases} \frac{x}{12}, & x \neq 1 \\ \frac{1}{4}, & x = 1 \end{cases},$$

for all $x \in X$.

Then
$$PST(X) \cup QAB(X) = [0,\ \tfrac{1}{12}] \cup [0,\ \tfrac{1}{24}] = [0,\ \tfrac{1}{12}] \subset [0,\ \tfrac{1}{6}] = ABST(X).$$

Clearly $AB = BA, PB = BP, TQ = QT, ST = TS, (AB)(ST) = (ST)(AB)$. and $A,\ B,\ S$ and T are continuous. Also the condition (1. 2) of Main result is satisfied and the pair $\{P,\ AB\}$ is compatible of type (β) if $lim_{n\to\infty}\ x_n = 0$ where $\{x_n\}$ is a sequence in X such that

$lim_{n\to\infty}\ Px_n = lim_{n\to\infty}\ ABx_n = 0$ for some $0 \in X$.

Similarly the pair $\{Q, ST\}$ is also compatible of type (β)

Thus all the conditions of main theorem are satisfied and also $0 \in X$ is the unique common fixed point of $A,\ B,\ S,\ T,\ P$ and Q.

References

[1] Alaca C. , Turkoglu D. and Yildiz C. , Fixed point in intuitionistic fuzzy metric spaces. Chaos solutions and fractals 2006-09: 1073-78.

[2] Cho Y. J. , Fixed point in fuzzy metric spaces. J. fuzzy Math. 1997; 5(4): 949-962.

[3] Cho Y. J. , Pathak H. K. , Kang S. M. and Jung J. S. , Common fixed points of compatible maps of type (β) on fuzzy metric spaces. Fuzzy sets Syst. 1998; 93: 99-111.

[4] Dubosis D. and Prade H. , Fuzzy sets: Theory and applications to policy, analysis and information systems. Newyork: Plenum press; 1980.

[5] George A. and Veeramani P. , On some results in fuzzy metric spaces. Fuzzy sets and systems 64(1994): 395-399.

[6] Grabiec M. , Fixed points in fuzzy metric space, fuzzy sets and systems 27(1988):

[7] Gregory V. , Romaguera S. and Veeramani P. , A note on intuitionistic fuzzy metric spaces. Chaos solutions & fractals
28(2006) : 902-905.