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MIN- NORM L-FUZZY INTERVAL VALUED SUBGROUPS OFNEAR RINGS

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ABSTRACT

In this paper we present the notion of Interval Min norm L –fuzzy R- subgroup of near rings and enquire some of their properties. Using Upper level set, we give a characterization of Min^i L-fuzzy right R-subgroup. Finally we build the ideal of the homomorphic inverse image.

Keywords : Interval number, Interval min norm, L –fuzzy set, Homomorphism, Upper level set, Near rings.

1.INTRODUCTION

The fuzzy set theory was initiated by L.A Zadeh in 1965. Since a membership in a fuzzy set theory is a matter of degree, we can represent the gradual membership of an element of a set describing the fuzzy parameters like cold, hot, tall, shot etc in a better way. Near rings were first studied by Fitting in 1932. It is a generalization of a ring. If in a ring we ignore commutativity of addition and one distributive law then we get a near ring.

Zadeh [7] made an extension of the concept of fuzzy sets by an interval valued set. That is a fuzzy set with an interval valued membership function. Interval valued fuzzy sets have many applications in several areas. A. Solaraju and R. Nagarajan introduced the concept of structures of L-fuzzy groups [3], [4] [5], [6].

In this paper we apply the notion of interval valued fuzzy sets to near rings. we present the notion of Interval Min norm L –fuzzy R- subgroup of near rings and enquire some of their properties. Using Upper level set, we give a characterization of Min^i L-fuzzy right R-subgroup. Finally we build the ideal of the homomorphic inverse image.

2. PRELIMINARIES

We first recall some basic concept which are used to present the paper.

DEFINITION 2.1

An interval number on [0, 1], say $\bar{a}$ is a closed subinterval of [0, 1], ie) $\bar{a} = [a^-, a^+]$ where $0 \leq a^- \leq a^+ \leq 1$.

For any interval number $\bar{a} = [a^-, a^+]$ and $\bar{b} = [b^-, b^+]$ on [0, 1], we define

1) $\bar{a} \leq \bar{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$.
2) $\bar{a} = \bar{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$.
3) $\bar{a} + \bar{b}$ = [$a^- + b^-$, $a^+ + b^+$], whenever $a^- + b^- \leq 1$ and $a^+ + b^+ \leq 1$.

DEFINITION 2.2

Let X be a set. A mapping $A: X \rightarrow [0,1]$ is called a fuzzy set in X. let A be a fuzzy set in X and $\alpha \in [0,1]$.

Define U(A: α) as follows.

U(A: $\alpha) = \{x \in X / A(x) \geq \alpha\}$. Then U(A: α) is called the upper level cut of A.

DEFINITION 2.3

Let X be a set. A mapping $\bar{A} : X \rightarrow D[0,1]$ is called on interval – valued fuzzy set (briefly i-v fuzzy set) of X, where $D[0,1]$ denotes the family of all closed subinterval of [0, 1], and $\bar{A}(x) = [A^-(x), A^+(x)]$, $\forall x \in X$. Where A^- and A^+ are fuzzy sets in X.

For an i-v fuzzy set $\bar{A}$ of a set X and $(\alpha, \beta) \in [0,1]$ define $U(\bar{A}: [\alpha, \beta])$ as follows.

$\bar{U}(A: [\alpha, \beta]) = \{x \in X / \bar{A}(x) \leq [\alpha, \beta]\}$. Which is called the level subset of $\bar{A}$.

DEFINITION 2.4

By a near ring, we mean a non empty set R with two binary operations '+' and '·' satisfying the following axioms :

i. $(R, +)$ is a group

ii. $(R, \cdot)$ is a semi- group

iii. $x \cdot (y + z) = x \cdot y + x \cdot z$ for all $x, y, z \in R$.

It is a left near - ring because it satisfies the left distributive law .

3. INTERVAL MIN – NORM L- FUZZY R- SUBGROUPS

The notion of an interval Min- Norm was introduced by jun and kim as follow.

DEFINITION 3.1

A mapping $min^i : D[0,1] \times D[0,1] \rightarrow D[0,1]$ given by $min^i(\bar{a}, \bar{b}) = [\min(a^-, b^-), \min(a^+, b^+)]$ $\forall\, \bar{a}, \bar{b} \in D[0,1]$ is called an interval Min-Norm.

PROPOSITION 3.2

Let min^i be an interval Min-Norm on $D[0,1]$ then

i) $min^i(\bar{a}, \bar{b}) = \bar{a}$, $\forall\, \bar{a} \in D[0,1]$

ii) $min^i(\bar{a}, \bar{b}) = min^i(\bar{b}, \bar{a})$ $\forall\, \bar{a}\, \bar{b} \in D[0,1]$

iii) If $\bar{a} \leq \bar{b}$ in $D[0,1]$, then $min^i(\bar{a}, \bar{c}) \leq min^i(\bar{b}, \bar{c})$, $\forall\, \bar{c} \in D[0,1]$

PROOF

Let min^i be an interval Min-Norm on $D[0,1]$ by the definition

A mapping $min^i : D[0,1] \times D[0,1] \rightarrow D[0,1]$ given by $min^i(\bar{a}, \bar{b}) = [\min(a^-, b^-), \min(a^+, b^+)]$ $\forall\, \bar{a}, \bar{b} \in D[0,1]$

i) $min^i(\bar{a}, \bar{b}) = \bar{a}$ $\forall\, \bar{a} \in D[0,1]$

Let $\bar{a} = [a^-, a^+]$ and $\bar{b} = [b^-, b^+]$ $\quad \bar{a}\,\bar{b} \in D[0,1]$

Consider,

$$min^i(\bar{a}, \bar{b}) = [\min(a^-, b^-),\ \min(a^+, b^+)]$$

$$= [\min(a^-, a^+),\ \min(b^-, b^+)]$$

$$= [\bar{a}, \bar{b}] \quad = \quad \bar{a}\,.\,\bar{a}$$

We know that $\bar{a} \le \bar{b}$ iff $a^- \le b^-$ and $a^+ \le b^+$

Hence $min^i(\bar{a}, \bar{b}) = \bar{a} \quad \forall\, \bar{a} \in D[0,1]$

ii) $min^i(\bar{a}, \bar{b}) = min^i(\bar{b}, \bar{a}) \quad \forall\, \bar{a}\,\bar{b} \in D[0,1]$

Consider,

$$min^i(\bar{a}, \bar{b}) = [\min(a^-, b^-),\ \min(a^+, b^+)]$$

$$= [\min(a^-, a^+),\ \min(b^-, b^+)]$$

$$min^i(\bar{a}, \bar{b}) = [\min(a^-, a^+),\ \min(b^-, b^+)] \quad \text{..........(1)}$$

$$min^i(\bar{b}, \bar{a}) = [\min(b^-, a^-),\ \min(b^+, a^+)]$$

$$= [\min(b^-, b^+),\ \min(a^-, a^+)]$$

$$min^i(\bar{b}, \bar{a}) = [\min(a^-, a^+),\ \min(b^-, b^+)] \quad \text{..........(2)}$$

From (1) and (2)

$$min^i(\bar{a}, \bar{b}) = min^i(\bar{b}, \bar{a}) \quad \forall\, \bar{a}\,\bar{b} \in D[0,1]$$

iii) $min^i(\bar{a}, \bar{c}) \le min^i(\bar{b}, \bar{c})$, $\forall\, \bar{c} \in D[0,1]$

Consider,

$$min^i(\bar{a}, \bar{c}) = [\min(a^-, c^-),\ \min(a^+, c^+)]$$

$= [\min(a^-, a^+),\ \min(c^-, c^+)]$

$\leq [\min(b^-, b^+),\ \min(c^-, c^+)]$

We know that if $\bar{a} \leq \bar{b}$ iff $a^- \leq b^-$ and $a^+ \leq b^+$

$\leq [\min(b^-, c^-),\ \min(b^+, c^+)]$

$\leq min^i(\bar{b}, \bar{c})$

Henc $min^i(\bar{a}, \bar{c}) \leq min^i(\bar{b}, \bar{c})$, $\forall\, \bar{c} \in D[0,1]$

DEFINITION 3.3

An interval valued fuzzy set $\bar{A}$ (i-v) is a near ring R is called an interval valued right (respectively left) R-subgroup of R with respect to the interval Min- Norm min^i (briefly a min^i L-fuzzy right (respective left)) R-subgroup if

(IVR_1) $\bar{A}(x - y, l) \geq min^i(\bar{A}(x, l), \bar{A}(y, l))$

(IVR_2) $\bar{A}(xr, l) \geq \bar{A}(x)$ (respectively)

$\bar{A}(rx) \geq \bar{A}(x)$ for all $x, y, r \in R$.

PROPOSITION 3.4

If $\bar{A}$ is a min^i L – fuzzy rights (respectively left) R-subgroups of a near ring R, then $\bar{A}(0, l) \geq \bar{A}(x, l)$ for all $x \epsilon R, l \epsilon L$

PROOF

For every $x \epsilon R, l \epsilon L$

$$\bar{A}(0, l) = \bar{A}(x - x, l) \geq min^i\{\bar{A}(x, l), \bar{A}(x, l)\}$$

$= [\min(A^-(x, l), A^-(x, l), \min((A^+(x, l), A^+(x, l))]$

$= [A^-(x, l), A^+(x, l)]$

$= [\bar{A}(x, l)]$

Hence $\bar{A}(0, l) \geq \bar{A}(x, l)$ for all $x \epsilon R, l \epsilon L$

NOTE

In what follows, the notion of L −fuzzy (Respectively min^i- fuzzy R subgroup means the notion of L −fuzzy (res− $min^i L$- fuzzy) right R − subgroup.

PROPOSITION 3.5

If $\{\bar{A}_i \,/\, i \epsilon \Lambda\}$ is a family of $min^i\ L$ −fuzzy R-subgroups of a near ring R then $\Lambda \bar{A}_i$ is a $min^i\ L$ −fuzzy R − subgroups of a near ring R. where Λ is any index set.

PROOF

Let $x, y \epsilon R, l \epsilon L$

we have, $(\Lambda \bar{A}_i)\ (x - y, l) = \inf\{\bar{A}_i(x - y, l) : i \epsilon \Lambda\}$

Assume that $\{\bar{A}_i \,/; i \epsilon \Lambda\}$ is a family of $min^i\ L$ −fuzzy R-subgroups of a near ring R .

Therefore

i) $\bar{A}_i\ (x - y, l) \geq\ min^i \overline{(A}(x, l) \leq \bar{A}(y, l))$

ii) $\bar{A}_i(xr, l) \geq \bar{A}(x)$

Consider $\wedge \bar{A}_i(x - y, l) = \inf\{\bar{A}_i(x - y, l); i\ \in \wedge\}$

$$\geq inf\{min^i\{\bar{A}_i(x, l), \bar{A}_i(y, l)\}; i\ \in \wedge\}$$

$$= min^i\left\{inf\{\bar{A}_i(x, l); i\ \in \wedge\}, inf\{\bar{A}_i(y, l); i\ \in \wedge\}\right\}$$

$$= min^i\{(\wedge \bar{A}_i)_{i \in \wedge}(x, l), (\wedge \bar{A}_i)_{i \in \wedge}(y, l)\}$$

Hence $\wedge \bar{A}_i(x - y, l) \geq min^i\{(\wedge \bar{A}_i)_{i \in \wedge}(x, l), (\wedge \bar{A}_i)_{i \in \wedge}(y, l)\}$

And for every $r, x \in R$ we have

$$\wedge \bar{A}_i(xr, l) = \inf\{\bar{A}_i(xr, l); i \in \wedge\}$$

$$\geq inf\{\bar{A}_i(xr, l); i \in \wedge\}$$

$$= \wedge \bar{A}_i(x, l)$$

Hence $\wedge \bar{A}_i(xr, l) \geq \wedge \bar{A}_i(x, l) \wedge \bar{A}_i(x, l)$

Hence $\wedge \bar{A}_i$ is a min^i L-fuzzy R subgroup of R.

PROPOSITION 3.6

Let R be a near- ring. An i-v L- fuzzy set $\bar{A}$ in R min^i L-fuzzy R subgroup of R if and only if A^+ and A^- are L- fuzzy R subgroup of R.

PROOF

Assume that A^+ and A^- are L- fuzzy R subgroup of R and let $x, y \in R$ and $l \in L$

To Prove an i-v L- fuzzy set $\bar{A}$ in R min^i L-fuzzy R subgroup of R.

i)$\bar{A}(x - y,\ l) \geq min^i(\bar{A}(x, l),\ \bar{A}(y, l))$

ii)$\bar{A}(xr,\ l) \geq \bar{A}(x, l)$

consider

$$\bar{A}(x - y,\ l) = [A^-(x - y, l), A^+(x - y, l)]$$

$$\geq min^i\{(A^-(x, l), A^-(y, l)\}, min^i\{(A^+(x, l), A^+(y, l)\}$$

$$= min^i(\bar{A}(x, l),\ \bar{A}(y, l))$$

Hence

$$\bar{A}(x - y,\ l) \geq min^i(\bar{A}(x, l),\ \bar{A}(y, l))$$

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Consider $\bar{A}\,(xr,\ l) = [A^-(xr,l), A^+(xr,l)]$

$$\geq [A^-(x,l), A^+(x,l)]$$

$$= \bar{A}\,(x,l)$$

Hence

$$\bar{A}\,(xr,\ l) \geq \bar{A}\,(x,l)$$

Hence $\bar{A}$ is a min^i L-fuzzy R- subgroup of R.

Conversely,

Assume that $\bar{A}$ are L- fuzzy R- subgroup of R.

To Prove

A^+ and A^- are L- fuzzy R- subgroup of R.

$$[A^-(x-y,l), A^+(x-y,l)] = \bar{A}\,(x-y,\ l)$$

$$\geq min^i\,(\bar{A}\,(x,l),\ \bar{A}\,(y,l))$$

$$= min^i[\{(A^-(x,l), A^+(x,l)\},\ \{(A^-(y,l), A^+(y,l)\}]$$

$$= min^i\,\{(A^-(x,l), A^-(y,l)\}, min^i\,\{(A^+(x,l), A^+(y,l)\}$$

It follows that,

$$\bar{A}\,(x-y,\ l)\ \geq\ min^i\,[A^-(x,l), A^-(y,l)]$$

And

$$\bar{A}\,(x-y,\ l)\ \geq\ min^i\,[A^+(x,l), A^+(y,l)]$$

For all $xr \in R$ $l \in L$ we've

$$[A^-(xr,l), A^+(xr,l)] = \bar{A}\,(xr,\ l)$$

$\geq \bar{A}(x, l)$

$= [A^-(x,l), A^+(x,l)]$

It follows that,

$A^-(xr,l) \geq A^-(x,l)$ and

$A^+(xr,l) \geq A^+(x,l)$

Hence A^- and A^+ are L- fuzzy R- subgroups of R.

PROPOSITION 3.7

Every R-subgroup of a near ring R can be realized as an upper level R subgroup of a min^i

L- fuzzy R- subgroup of R.

PROOF

Let H be an R subgroup of a near ring R and let $\bar{A}$ be on i-v L-fuzzy set in R defined by

$$\bar{A}(x,l) = \begin{cases} \bar{a} & if\ x \in H \\ \bar{0} & otherwise \end{cases}$$

Where $\bar{a}(\neq \bar{0}) \in D[0,1]$.

It is clear that $\bar{U}(\bar{A}, \bar{a}) = H$.

We will show that Ais a min^i L-fuzzy Rsubgroup of R.

Case (i)

If $, y \in H$, then $x - y \in H$ and so $\bar{A}(x-y,l) = \bar{a} = min^i(\bar{a}, \bar{a}) = min^i(\bar{A}(x,l), \bar{A}(y,l))$

Case (ii)

If $x, y \notin H$ then $\bar{A}(x,l) = \bar{0} = \bar{A}(y,l)$ and thus

$\bar{A}(x-y,l) \geq \bar{0} = min^i(\bar{0}, \bar{0}) = min^i(\bar{A}(x,l), \bar{A}(y,l))$

Suppose that the only one of x, y belongs to H say x. Then,

$$\bar{A}(x-y,l) \geq \bar{0} = min^{i}(\bar{a}, \bar{0}) = min^{i}(\bar{A}(x,l), \bar{A}(y,l))$$

Now if $x \in R/_H$ then $\bar{A}(x,l) = \bar{0}$ and so

$\bar{A}(xr,l) \geq \bar{0} = \bar{A}(x)$ for all $r \in R, \quad l \in L.$ If $x \in H$ then $xr \in H$ which implies that

$\bar{A}(xr,l) = \bar{a} = \bar{A}(x,l)$ for all $r \in R, \quad l \in L.$

Hence $\bar{A}$ is a min^{i} L-fuzzy R- subgroups of R.

PROPOSITION 3.8

Let H be a subset of a near ring R. A function $\psi_H(x,l) = \begin{cases} \bar{1} & if\ x \in H \\ \bar{0} & otherwise \end{cases}$

For all $r \in R, \quad l \in L$ is a min^{i} L- fuzzy R subgroup of R. if and only if H is an R- subgroup of R.

PROOF

Let H be an R- subgroup.

To prove $\psi_H(x,l)$ is a min^{i} L- fuzzy R subgroup of R.

Let R be a near ring. An i-v L-fuzzy set $\bar{A}$ in R min^{i} L- fuzzy R subgroup of R if and only if A^{-} and A^{+} are L-fuzzy R-subgroup of R.

Hence ψ_H is a min^{i} L- fuzzy R subgroup of R.

Conversely

Suppose that ψ_H is a min^{i} L- fuzzy R subgroup of R.

To prove

H be a subset of a near ring R. let $x, y \in H$ then $\psi_H(x,l) = \bar{1} = \psi_H(y,l)$ and so

$$\psi_H(x-y,l) \geq min^{i}\{\ \psi_H(x,l),\ \psi_H(y,l)\}$$

$= min^i(\bar{1},\bar{1}) = \bar{1}$

It follows that $\psi_H(x-y,l) = \bar{1}$ so that $x - y \in H$

Let $r \in R, l \in L$. Then we have $\psi_H(xr,l) \geq \psi_H(x,l) = \bar{1}$ and so $xr \in H$.Hence H is an R-subgroup of R.

PROPOSITION 3.9

If $\bar{A}$ is a min^i L-fuzzy R-subgroup of R then the set

$R_{\bar{A}} = \{x \in R \,/\, \bar{A}(x,l) = \bar{A}(0,l)\}$ is an R subgroup of R.

PROOF

Let $x, y \in R_{\bar{A}}$ and $l \in L$ then $\bar{A}(x,l) = \bar{A}(0,l) = \bar{A}(y,l)$ and so

$\bar{A}\,(x-y,l) \geq min^i\,(\bar{A}\,(x,l), \bar{A}\,(y,l))$

$\geq min^i\,(\bar{A}\,(0,l), \bar{A}\,(0,l))$

$\geq (\bar{A}\,(0,l)$

i.e. $\bar{A}\,(x-y,l) \geq \bar{A}\,(0,l))$

$\bar{A}\,(xr,l) \geq (\bar{A}\,(x,l) = \bar{A}\,(0,l))$ and so $x,r \in R_{\bar{A}}$

Hence $R_{\bar{A}} = \{x \in R \,/\, \bar{A}(x,l) = \bar{A}(0,l)\}$ is an R subgroup of R.

DEFINITION 3.10

Let R and R^1 be near rings. A map $f: R \to R^1$ is called a (near ring) homomorphism if $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in R$.

Let X and Y be sets. A mapping $f: X \to Y$ induces two mappings.

$F_f: IF(x) \to IF(y)$ and $F_{f^{-1}}: IF(x) \to IF(y)$.

We define $F_f(\bar{A})(y) = \begin{cases} sup\bar{A}(x)\ f^{-1}(y) \neq 0 \\ x \in f^{-1}(y) \\ 0\ otherwise \end{cases}$

And $F_f^{-1}(\bar{B})(y) = \bar{B}f(x)\ for\ all\ x \in X\ where\ \bar{A} \in IF(x).\bar{B} \in IF(y)$ And $f^{-1}(y) = \{x \in X\ /\ f(x) = y\}$.

PROPOSITION 3.11

Let $f: R \to R^1$ be a homomorphismof near rings. If $\bar{B}$ is a min^i L-fuzzy R-subgroup of R, then $F_f^{-1}(\bar{B})$ is a min^i L-fuzzy R subgroup of R.

PROOF

Let $x, y \in R$, then $F_f^{-1}(\bar{B})(x-y,l) = \bar{B}\big(f(x-y,l)\big)$

$$= \bar{B}\big(f(x,l) - f(y,l)\big) \geq min^i\left(\bar{B}\big(f(x,l)\big), \bar{B}\big(f(y,l)\big)\right)$$

$$= min^i\left(F_f^{-1}(\bar{B})(x,l), F_f^{-1}(\bar{B})(y,l)\right)$$

Let $r \in R, l \in L$ then we have

$$F_f^{-1}(\bar{B})(xr,l) = \bar{B}\big(f(xr,l)\big) = \bar{B}\big(f(x,l), f(r,l)\big)$$

$$\geq \bar{B}\big(f(x,l)\big) = F_f^{-1}(\bar{B})(x,l)$$

Hence $F_f^{-1}(\bar{B})$ is a min^i L-fuzzy R subgroup of R.

4.CONCLUSION

Y.B. Jun [2] introduced the concept of interval valued fuzzy R-subgroup of near ring. In this paper we present the notion of Interval Min norm L –fuzzy R- subgroup of near rings and enquire some of their properties. Using Upper level set, we give a characterization of Min^i L-fuzzy right R-subgroup. Finally we build the ideal of the homomorphic inverse image.

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