IJARSE
ISSN: 2319-8354

Derivation of Condition between in-variants $\mathrm{I}_1, \mathrm{I}_2$ and I_3 for equal Principal Stresses by using concept of Linear Algebra

Manjeel Kumar

Department of Mathematics, DAV College, Hoshiarpur

ABSTRACT

Principal stress is one kind of maximum or minimum normal stress when the corresponding shear stresses are zero, the plane on which principal stress act is known as principal plane. By the maximum principal stress theory, a brittle material ruptures when the maximum principal stress in the specimen reaches some limiting value for the material. This critical value can be inferred as the tensile strength measured using a uni-axial tension test. In practice, this theory is simple, but can only be used for brittle materials. So Principal stresses at a given point in a stressed body is vitally used in design information. With the help of it we can predict whether the design is suitable to hold a given load at a suitable point or not. Principal stresses and principal planes form a backbone of material stress analysis. As we know that there are infinite numbers of planes passing through the given point and the normal stress on each plane will be different from the other. Therefore If we know all the stress components $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}, \tau_{yx}, \tau_{zy}$ and τ_{xz} for a particular plane then the principal stresses are the roots of the characteristic equation given by

$$\sigma^3 - \sigma^2 I_1 + \sigma I_2 - I_3 = 0$$

Where I_1, I_2 and I_3 are stress in-variants. Major aspect of this paper is to discuss the condition in terms of stress in-variants of equal principal stresses along the co-ordinate axes by taking principal axes as the axes of reference.

Keywords: Characteristic equation, Eigen values, Hydrostatic stress, Principal stress, Strain.

1. INTRODUCTION

There can be infinite number of planes passing through a point and normal stress on each plane will be different from one another. There will be only one plane where normal stress is maximum that plane is known as principal plane and the normal stress on this plane is known as principal stress. Also there is one more plane where normal stress value is minimum, this plane is also known as principal plane and normal stress is known as principal stress.

2. BASIC TERMINOLOGY

For the three- dimensional stress analysis, there are three components of normal stresses denoted as $\sigma_x, \sigma_y, \sigma_z$ while there are six shear stress components denoted as $\tau_{xy}, \tau_{yz}, \tau_{zx}, \tau_{yx}, \tau_{zy}$ *and* τ_{xz} as shown in the Fig.2.1. There are three planes of zero shear stress exist, that these planes are mutually perpendicular, and that on these planes the normal stresses have maximum or minimum values.

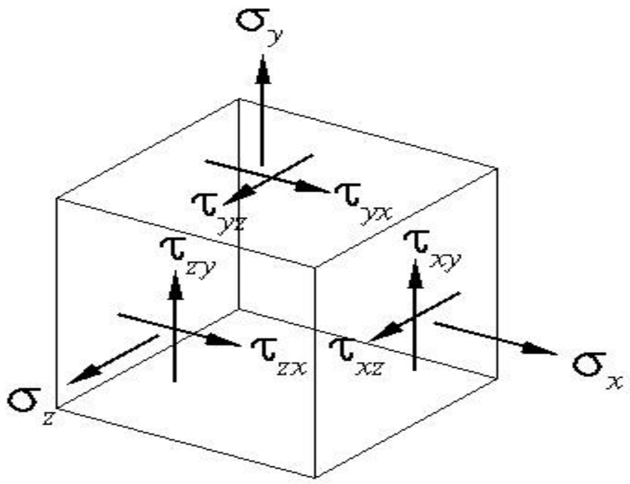

Fig.2.1

As the stress components are the roots of the characteristic equation

$$\begin{vmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma \end{vmatrix} = 0.$$

$$\text{Or } \sigma^3 - \sigma^2 I_1 + \sigma I_2 - I_3 = 0 \qquad \dots\dots(i)$$

$$\text{Where } I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy} - \tau_{yz} - \tau_{zx}$$

$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2$$

3. DERIVATION OF CONDITION

By considering the planes of zero shear stress as the coordinate planes and normal to them as the coordinate axes which are the directions corresponding to optimal normal stress. Now, all the shear components become zero i.e. $\tau_{xy} = \tau_{yz} = \tau_{zx} = \tau_{yx} = \tau_{zy} = \tau_{xz} = 0$.

Therefore above expression reduced to

$$\mathrm{I}_1 = \sigma_x + \sigma_y + \sigma_z \qquad \ldots\ldots(ii)$$

$$\mathrm{I}_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x \qquad \ldots\ldots(\boldsymbol{iii})$$

$$\mathrm{I}_3 = \sigma_x\sigma_y\sigma_z \qquad \ldots\ldots(iv)$$

Now by suitable transformation we can reduced the eq.(1) to another equation in which the second degree terms are not present. For that we make a substitution as

$$\sigma' = \sigma - \frac{I_1}{3} \qquad \ldots\ldots(v)$$

Equation (1) reduced to the form

$$\sigma'^3 + 3A\sigma' + B = 0 \qquad \ldots\ldots(vi)$$

$$Where \quad A = \frac{9I_1I_2 - 27I_3 - 2I_1^3}{27}, \; B = \frac{3I_2 - I_1^2}{9}$$

$$A = \frac{9(\sigma_x + \sigma_y + \sigma_z)(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x) - 27(\sigma_x\sigma_y\sigma_z) - 2(\sigma_x + \sigma_y + \sigma_z)^3}{27}$$

$$= \frac{-\left\{(\sigma_x - \sigma_y)^3 + (\sigma_y - \sigma_z)^3 + (\sigma_z - \sigma_x)^3\right\}}{27} \qquad \ldots\ldots(vii)$$

$$\mathrm{B} = \frac{3(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x) - (\sigma_x + \sigma_y + \sigma_z)^2}{9}$$

$$= \frac{-\left\{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right\}}{18} \qquad \ldots\ldots(viii)$$

For equal principal stresses i.e. $\sigma_x = \sigma_y = \sigma_z$, we get from equation (vii) and (viii).

$$A = 0 \; and \; B = 0$$

$$9I_1I_2 - 27I_3 - 2I_1^3 = 0 \; and \; 3I_2 - I_1^2 = 0$$

$$\Rightarrow \quad 27I_3 = I_1^3$$

$$Hence\ the\ condition\ for\ equal\ Principal\ stresses\ is$$

$$27I_3 = I_1^3$$

IV CONCLUSION

When the three principal stresses defining a triaxial state of stress become equal i.e. $\sigma_x = \sigma_y = \sigma_z$ defining a hydrostatic state of stress and no shear stress developed. For this state of hydrostatic stress no plastic deformation occurs. Therefore above condition is very useful for the study of plastic deformation.

REFERENCES

1 P D S Verma, Theory of Elasticity ,Vikas Publishing House,1997

2. Warren C. Young, Richard G BudynasRoark's Formulas for Stress and Strain, McGraw Hill Professional, 04-Oct-2001

3. Srinath , Advanced Mechanics Of Solids, Tata McGraw Hill Education

4. Hani M. Tawancy, Anwar Ul-Hamid, Nureddin M. Abbas, Practical Engineering Failure Analysis 5.S.P. Timoshenko, J.N. Goodier, Theory of Elasticity, Tata McGraw Hill

6. Teodor M. Atanackovic, Ardeshir Guran Theory of Elasticity for Scientists and Engineers, Springer Science & Business Media

7. A.I. Lurie Theory of Elasticity, Springer

8. R. J. Atkin, N. Fox An Introduction to the Theory of Elasticity, Dover Publication INC Mineola , New York.

9. Richard B. Hetnarski, Józef Ignaczak ,The Mathematical Theory of Elasticity, Second Edition, CRC Press

10. L. M. Kachanov,Fundamentals of the Theory of Plasticity, Dover Publication INC Mineola , New York.