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Spin determination of ^{104}Pd and ^{105}Pd superdeformed bands through VMI model

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Abstract.

VMI model has been applied to accredit the band head spin (I_0) of ^{104}Pd and ^{105}Pd superdeformed (SD) rotational bands. The best fit method (BFM) is taken into the account to achieve the band head spin (I_0) of ^{104}Pd and ^{105}Pd SD bands. The parameters of the VMI model are used to calculate the root mean square deviation values among the calculated and the observed transition energies. The computed and the observed transition energies show a good agreement with each other. Hence, the VMI model is a powerful tool for assigning the band head spin (I_0) of ^{104}Pd and ^{105}Pd SD bands.

***Keywords*: Band head spin, best fit method, superdeformed bands, transition energies, VMI model.**

1. INTRODUCTION

The phenomenon of high spin superdeformed (SD) rotational bands was firstly observed by Twin et el. [1] in ^{152} Dy(1) nucleus. The different detector array helps to establish SD rotational bands in various A~190, 150, 130, 80 and 60 mass regions. The identification of the SD bands in different mass regions can be done through its deformation parameters and life time measurements. However, the only known information for the SD rotational bands is their intraband E2 transition energies and intensities. Because of the unknown transition linking among the SD bands and normal deformed (ND) bands many attributes like spin prediction, exact excitation energies and parities remain uninvestigated. To explore, the physics of the SD rotational bands the level spin prediction is an important concept. The uncertainty in the spin prediction advances to study the quantity called as dynamic moment of inertia ($J^{(2)}$) for the SD bands. To deepen the understanding of the SD rotational bands the knowledge of kinematic moment of inertia ($J^{(1)}$) and the dynamic moment of inertia ($J^{(2)}$) is crucial. Subsequently, the systematic study of ($J^{(1)}$) and ($J^{(2)}$) with N_PN_n was studied by Sharma et al. [2]. The ($J^{(2)}$) show an increment with rise in rotational frequency for A~ 190 mass region [3] and an increment with a decrement in rotational frequency for A~150, 130 mass regions [4-5].

The description of SD rotational bands in A~ 60 mass region was illustrated by Xin and Meng [6]. Moreover, the classification of SD rotational bands in A~60 mass region was depicted by Anderson et al. [7]. Cranked relativistic mean field theory and Nilsson approach was reported by Afanasjev et al. [8] to explain the highly and SD bands in A~60 mass region. Svensson et al. [9] reported the first SD rotational bands in A~60 mass region. In Ref. [9] quadrupole moment was found to be 2.7eb. The concept of double magic SD rotational bands in Zn nucleus was studied by Svennson et al. [10]. It was also identified in Ref. [10] that the transition linking of SD bands to yrast bands provides spin, parity and excitation energies. The first SD rotational band in A~80 mass region was reported by Baktash et al. [11]. The identification of SD rotational bands in A~80 mass region can be acknowledge through its highly collective nature, large moment of inertia and average transition quadrupole moment. Many SD rotational bands are detected with the help of detector arrays in A~80 mass region i. e. in $^{81\text{-}82}$Sr, $^{82\text{-}84}$Y, $^{83\text{-}84}$Zr, ^{86}Zr and ^{87}Nb [12-17]. The shape coexistence effect was studied by Dudek et al. [18].The shape evolution in SD rotational bands in A~80-90 mass regions were studied by Lagergren et al. [19].

The isospectral SD rotational bands in ^{88}Mo and ^{89}Tc were studied by Lagergren et al. [20]. The level spin of superdeformed bands in A~80 mass region was calculated by Dadwal et al. [21]. Even the band head spin assignment in A~60- 80 mass region through nuclear softness formula was determined by Sharma and Mittal [22]. Apart from A~60, 80 and 90 mass regions, A~100 mass region is of special interest. It is because the phenomenon of superdeformation was come into the play in ^{104}Pd and ^{105}Pd. The phenomenon of superdeformation in ^{104}Pd and ^{105}Pd was studied by Macchiavelli [23]. It was depicted in Ref. [23] that the band in ^{104}Pd and ^{105}Pd comprises of E2 transitions. Their regular cascades and calculated moments of inertia proposed that these bands occupies the superdeformed configuration (ε-0.35-0.4), comparable to those recognized in the Ce-Nd region. More confirmations to this regard is provided by calculating the value of quadrupole moment and lifetime measurements. The abrupt increment in $J^{(2)}$ in the SD band of ^{105}Pd at the highest spins may be due to the arrangement of a pair of $i_{13/2}$ neutrons. However, if the $A^{5/3}$ mass dependence is removed then the ($J^{(1)}$) and ($J^{(2)}$) of ^{104}Pd and ^{105}Pd bands are identical to those calculated in the Ce-Nd region. To attain the band head spin many methods are brought into the limelight like Harris 2 parameter, Lipas-Eijiri, Suq(2) model, VMI model etc [24-28]. Therefore, in this present paper we have calculated the band head spin for ^{104}Pd and ^{105}Pd SD bands by using the VMI model.

II. FORMALISM

VMI model was developed by Marriscotti et al. [29]. In this model the concept of sum of total energy i.e. the potential energy term and the rotational energy term was used to explain the energy levels with angular moment (I). To define each nucleus the two parameters of VMI model, that is the band head moment of inertia (J_0) and the restoring force constant (C) is taken into the account.

$$E_I(J) = \frac{1}{2}C(J_I - J_0)^2 + \frac{1}{2}\left[\frac{I(I+1)}{J_I}\right] \tag{1}$$

The band head energy of the SD rotational bands may be suggested as

$$E_I(J) = E_0 + \frac{1}{2J}[I(I+1) - I_0(I_0+1)] + \frac{1}{2}C(J_I - J_0)^2 \tag{2}$$

E_0 and J_0 are the band head energy and ground moment of inertia respectively. When an equilibrium condition is maintained, the moment of inertia may be calculated as

$$\frac{\partial E(J_I)}{\partial J} = 0 \tag{3}$$

which gives

$$J_I^3 - J^2 J - [I(I+1) - I_0(I_0+1)]/2C = 0 \tag{4}$$

As equation (4) has a real root for J_0 and C. Therefore, joining equation (2) and (4) we get

$$E_I = E_0 + \frac{[I(I+1) - I_0(I_0+1)]}{2J_0} \times \left[1 + \frac{I(I+1) - I_0(I_0+1)}{4C{J_0}^3}\right] \tag{5}$$

For SD bands the only available knowledge is the E_γ transition energies so, SD bands can be determined as

$$E_\gamma(I) = E(I) - E(I-2) \tag{6}$$

Using equation (5) and (6) we get

$$E_\gamma(I \to I-2) = \frac{[I(I+1)-(I-2)(I-1)]}{2J_0} + \frac{[I(I+1)]^2 - [(I-2)(I-1)]^2}{8C{J_0}^4} \tag{7}$$

where J_0 and C are the parameters which can be obtained by the technique of fitting.

III. RESULTS AND DISCUSSION

VMI model is used to achieve the band head spin of ^{104}Pd and ^{105}Pd SD bands of A~100 mass region. The least square fitting approach was taken into the account to fit the recognized experimental transition energies in equation (7). A data has been taken from the tables of SD bands and fission isomers given by Singh et al. [30]. A contrast is made among the determined and noticed transition energies. The determined and noticed transitions are in accordance with each other whenever a definite spins are accredited. However, a small discrepancy of ±1 in the values of I_0 exhibits an appreciable advance in the root mean square (rms) deviation. The rms deviation may be illustrate as

$$\chi = \left[\frac{1}{n}\Sigma\left(\frac{\mathrm{E}_\gamma^{\mathrm{cal}}(\mathrm{I_i}) - \mathrm{E}_\gamma^{\mathrm{exp}}(\mathrm{I_i})}{\mathrm{E}_\gamma^{\mathrm{exp}}(\mathrm{I_i})}\right)^2\right]^{1/2} \tag{8}$$

The band head spin of ^{104}Pd and ^{105}Pd SD bands in A~100 mass region attained by VMI model is shown in Table 1. It is examined from Table 1 that I_0 of ^{104}Pd and ^{105}Pd SD bands obtained by VMI model is in accordance with the experimental data (see Table 1). More verification in this regard is given by χ plot showing the band head spin of ^{104}Pd and ^{105}Pd SD bands. Even the computed energy transition values and experimental transition energies are in accordance with each other (see Table 2-3).

Table 1: The band head spin (I_0) obtained for ^{104}Pd and ^{105}Pd SD bands by using VMI model.

SD bands	$E_\gamma(I_0+2 \to I_0)$	VMI model	Experimental
^{104}Pd	1263	23	24
^{105}Pd	1209	21.5	21.5

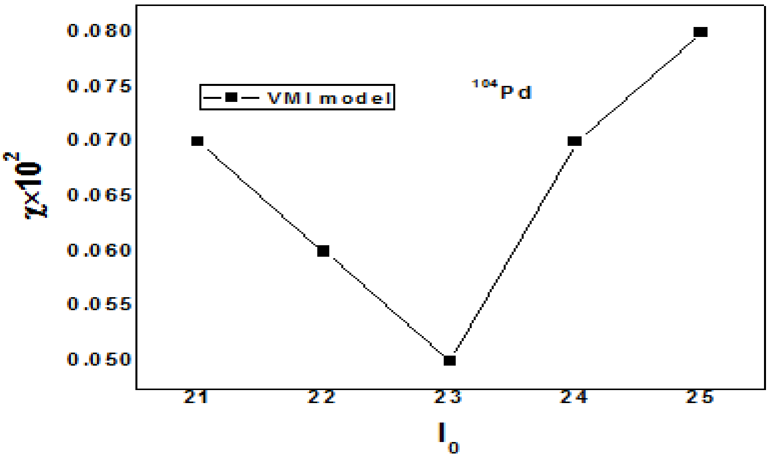

Fig. 1. χ plot to obtain I_0 for ^{104}Pd SD bands.

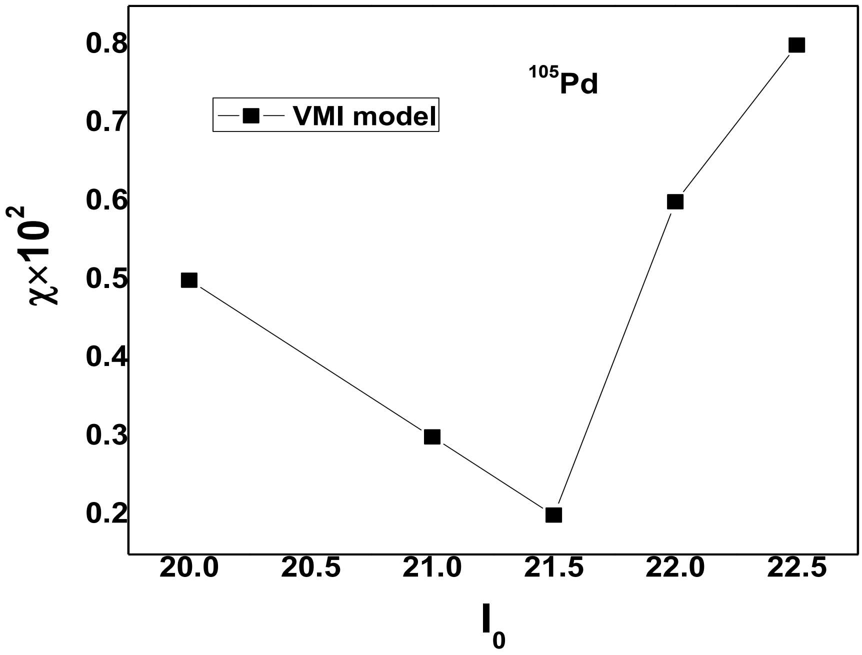

Fig. 2. χ plot to obtain I_0 for ^{105}Pd SD bands.

Table. 2 Spin determination of ^{104}Pd SD bands using VMI model. I_0 corresponds to the band head spin. δ is a difference between experimental and calculated transition energies.

$E_\gamma^{exp}(I)$	$I_0 = 22$			$I_0 = 23$			$I_0 = 24$		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
1263	24	1257.7	5.3	25	1256.1	6.9	26	1259.2	3.8
1381	26	1380.3	0.7	27	1380.0	1	28	1380.5	0.5
1511	28	1507.4	3.6	29	1508.1	2.9	30	1506.9	4.1
1638	30	1639.7	-1.7	31	1640.7	-2.7	32	1638.8	-0.8
1763	32	1777.3	-14.3	33	1778.2	-15.2	34	1776.5	-13.5
1919	34	1920.6	-1.6	35	1920.8	-1.8	36	1920.4	-1.4
2079	36	2070.0	9.0	37	2068.9	10.1	38	2071.0	8.0

Table. 3 Spin determination of ^{104}Pd SD bands using VMI model. I_0 corresponds to the band head spin. δ is a difference between experimental and calculated transition energies.

$E_\gamma^{exp}(I)$	$I_0 = 21$			$I_0 = 21.5$			$I_0 = 22$		
	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ	I	$E_\gamma^{cal}(I)$	δ
1209	23	1163.6	45.4	23.5	1165.6	43.4	24	1167.5	41.5
1282	25	1274.7	7.3	25.5	1275.6	6.4	26	1276.5	5.5
1379	27	1387.7	-8.7	27.5	1387.8	0.2	28	1387.9	-8.9
1488	29	1502.8	-14.8	29.5	1502.3	-14.3	30	1501.8	-13.8
1597	31	1620.3	-23.3	31.5	1619.3	-22.3	32	1618.5	-21.5
1720	33	1740.1	-20.1	33.5	1739.0	-19.0	34	1738.1	-18.1
1846	35	1862.5	-16.5	35.5	1861.6	-15.6	36	1860.7	-14.7
2007	37	1987.7	19.3	37.5	1987.2	19.8	38	1986.7	20.3
2141	39	2115.7	25.3	39.5	2116.0	25	40	2116.2	24.8
2240	41	2246.9	-6.9	41.5	2248.1	-8.1	42	2249.3	-9.3

IV. CONCLUSION

In this present work, the band head spin of ^{104}Pd and ^{105}Pd SD bands is obtained by VMI model. It is noticed that the obtained band head spin (I_0) of ^{104}Pd and ^{105}Pd SD bands are in accordance with the experimental data. Even, the computed and the noticed experimental transition energies are also in accordance with each other. Hence, the VMI model works efficiently for achieving the band head spin (I_0) for ^{104}Pd and ^{105}Pd SD bands of A~ 100 mass region.

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