

Mean Deviation a Tool for Solving Transportation Problem

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Abstract

There are several methods for obtaining initial basic feasible solution (IBFS) of transportation problem (TP) such as north west corner(NWC) rule, least cost method (LCM) and Vogel approximation method (VAM) etc. In this paper we have used the concept of mean deviation to solve transportation problem where feasible solution are near to optimal solution and in many cases it coincides with optimal solution. Numerical examples are also given in support of the result.

Keywords: Transportation Problem ; Feasible Solution ; Optimal solution .

1. Introduction

Transportation problem was first formulated in 1941 by Hitchcock [3], is a special case of linear programming problem in which our objective is to satisfy the demand at destinations from the supply at the minimum transportation cost. It was further developed in 1949 by Koopman [5] and in 1951 by Dantzig [2].

A certain class of linear programming problem known as transportation problems arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the contexts of determining optimum shipping pattern. For example: A product may be transported from factories to retail stores. The factories are the sources and the stores are the destinations. The amount of products that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destination have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of m origin and it is required that the given quantities of the product be shipped to each of n destinations. The minimum cost of shipping a unit of the product from any origin to any destination is known. The shipping schedule which minimizes the total cost of shipment is to be determined. The problem can be formulated as:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \dots, n$$

$$x_{ij} \geq 0, \text{ for all } i, j$$

For each supply point $i, (i = 1, 2, \dots, m)$ and demand point $j, (j = 1, 2, \dots, n)$

c_{ij} =unit transportation cost from i^{th} source to j^{th} destination

x_{ij} =amount of homogeneous product transported from i^{th} source to j^{th} destination

a_i =amount of supply at i^{th} source.

b_j =amount of demand at j^{th} destination.

where a_i and b_j are given non-negative numbers and assumed that total supply is equal to total demand, i.e. $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$, then transportation problem is called balanced otherwise it is called unbalanced.

The aim is to minimize the objective function satisfying the above mentioned constraints. In the classical transportation problem of linear programming, the traditional objective is one of minimizing the total cost.

Many authors have tried to solve the problems in different ways to reach the optimality in minimum number of steps. In 1990, Kirca and Satir [4], have explained about a heuristic for obtaining an initial solution for the TP. They explained about the superior performance of the new heuristic over VAM in terms of total cost obtained, number of iterations required to reach the final solution.

In 2012 Qudoods et al, [7], have discussed ASM-method to find optimal solution of wide number of problems. In 2014, Mamidi and Murphy [6], have tried to reveal that proposed direct methods namely NMD and Exponential method do not present optimal solution at all times. In 2014, Babu et al, [1], have proposed a new algorithm named implied cost method where feasible solutions are lower than VAM and very close to optimal solution. After studying the literature we have proposed DSM method to find the initial feasible solution to transportation problems.

Because of the special structure of the transportation model, the problem can also be represented as Table 1

Table 1: Tabular representation of model (α)

Destination \rightarrow source \downarrow	D_1	D_2	...	D_n	supply(a_i)
S_1	c_{11}	c_{12}	...	c_{1n}	a_1
S_2	c_{21}	c_{22}	...	c_{2n}	a_2
\vdots	\vdots	\vdots	...	\vdots	\vdots
S_m	c_{m1}	c_{m2}	...	c_{mn}	a_m
Demand (b_j)	b_1	b_2	...	b_n	



1.1. definitions

1. **Mean Deviation**:- Mean deviation of n observations x_1, x_2, \dots, x_n about any arbitrary real no, A is given by

$$M.D)_A = 1/n \sum_{i=1}^n |x_i - A|$$

2. **Feasible Solution** : - Feasible solution of a transportation problem is that solution for which all $x_{ij} \geq 0$.

3. **Basic Feasible Solution** :- A feasible solution of a transportation problem is said to be basic feasible if total no. of allocations is $m+n-1$.

4. **Optimal Solution** :- Solution of transportation problem which cannot be further improved is called Optimal Solution .

1.2. Algorithm of Proposed Method

To proceed with proposed method the given steps are followed:

step 1. Represent the given TP into the form of cost matrix as Table 1

step 2. If the TP is not balanced than make it balance by adding dummy row or column as per necessity.

step 3. Find Mean Deviation of each row about its least cost.

step 4. Find Mean Deviation of each column about its least cost.

step 5. Allocate the amount (minimum of supply and demand) at the smallest cost in the row or column where mean deviation is maximum .

step 6. If tie occur at step 5 then first do allocation at the place where cost is least .

step 7. Cross out the row or column which ever is satisfied in step 5.

step 8. For the uncrossed rows and columns repeat the step 3 and 4 and do the next allocation as in step 5 and 6. Continue in this way until each row and column is satisfied.

2. Numerical Examples

Numerical example: Input data and initial solution obtained by applying DSM method for different examples is given in tables 2-4.



Table 2: Input data and initial solution

Ex.	Input Data	Obtained Allocations by mean deviation	Obtained Cost	optimal solution
1	$[c_{ij}]_{3 \times 3} = [6 \ 4 \ 3; 2 \ 1 \ 6; 4 \ 8 \ 3]; [a_i]_{3 \times 1} = [30, 40, 50]; [b_j]_{1 \times 3} = [60, 25, 35]$	$x_{13} = 30, x_{21} = 15, x_{23} = 25, x_{31} = 45, x_{33} = 5$	340	340

Table 3: Input data and initial solution

Ex.	Input Data	Obtained Allocations by mean deviation	Obtained Cost	optimal solution
1	$[c_{ij}]_{3 \times 3} = [17 \ 20 \ 23; 26 \ 20 \ 26; 25 \ 17 \ 28]; [a_i]_{3 \times 1} = [34, 52, 51]; [b_j]_{1 \times 3} = [51, 40, 46]$	$x_{11} = 34, x_{21} = 6, x_{23} = 46, x_{31} = 11, x_{32} = 40$	2885	2885

Table 4: Input data and initial solution

Ex.	Input Data	Obtained Allocations by mean deviation	Obtained Cost	optimal solution
1	$[c_{ij}]_{3 \times 3} = [12 \ 8 \ 6; 4 \ 2 \ 12; 8 \ 16 \ 6]; [a_i]_{3 \times 1} = [20, 30, 40]; [b_j]_{1 \times 3} = [50, 15, 25]$	$x_{13} = 20, x_{21} = 15, x_{22} = 15, x_{31} = 35, x_{33} = 5$	520	520

3. Conclusions

Thus we have concluded that our method is easy to understand and apply as the initial solution is nearly equal to the optimal solution and in many cases it is equal to optimal solution.

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