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Improved approaches to obtain better initial basic feasible solution of Transportation Problem

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Abstract

This paper presents a note on S. K. Goyal's view point, which was given by him for improving 'total opportunity cost method' (TOM) and JHM method. In the note on S. K. Goyal's view point, we have discussed two cases in which Goyal's modified TOM not gives better solution than TOM and further improved modified TOM by changing the rule of making allocations, which gives better solution than TOM and modified TOM. Also note on JHM method is completion of JHM method for obtaining best initial feasible solution of transportation problem when tie occurs in selecting least cost cell for assigning respective demand quantities. After this improvement, JHM gives more efficient solution. .
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1. Introduction

For the organizations, it is very important to transporting goods from numerous sources to different destinations in minimal possible cost of transportation. So that profit should be maximum. For this purpose, researchers have developed number of methods to find minimal transportation cost. For solving transportation problem (TP), start with a initial basic feasible solution is very important which should be efficient and nearer to optimal solution. So for this purpose, huge techniques are available in literature to find better initial basic feasible solution, which are, North west corner method, Best cell method, Vogel's approximation method (VAM) [12], Shimshak et al.'s modified VAM [14], Improved VAM by Goyal [3], Improved VAM by Ramakrishnam [11], Balakrishnan's modified VAM [1], Russel's approximation method [13], Greedy heuristic [16], Maximum demand method [10], Total opportunity cost method (TOM) [7], Improved version of TOM [4], TOCM-VAM [9], Extreme difference method [6], Cost sum method [8], Zero suffix method [15], JHM method [5] etc. But our main purpose is to further improve S. K. Goyal 's [4] modified TOM and JHM method [5], so that better initial basic feasible solution can be obtained for TP. So in section 2 and section 3 we will discuss about these both methods.

2. Note on S. K. Goyal's view point

As Kirca and Satir [7] obtained a heuristic named total opportunity cost method (TOM) in which first they determined row opportunity cost of each cell by subtracting the minimum cost of corresponding row from each unit transportation cost and column opportunity cost of each cell by subtracting the minimum cost of corresponding column from each unit transportation cost. Then calculate total opportunity cost of each cell by adding corresponding row opportunity cost and column opportunity cost and made allocations using best cell method. Then S. K. Goyal [4] had modified Kirca and Satir [7] approach by taking opportunity cost as maximum of row opportunity and column opportunity cost and then for making allocations he had applied the best cell method .But this does not always gives better solution only because the method applied for giving allocations is same as Kirca and Satir [7]. There are two reasons behind that which are explained below:

Case 1

In S. K. Goyal's [4] approach, In some cases opportunity cost of a cell with high unit transportation cost is less than the opportunity cost of a cell with low unit transportation cost comparative to the Kirca and Satir [7] approach because they calculate opportunity cost as the sum of row and column opportunity cost. So in these cases allocation is made in the cell with high unit transportation cost (TC) by using least cost method which leads to high transportation cost than Kirca and Satir [7]. Counter example to describe case 1 is given in table 1 and results obtained by applying both existing techniques to find IBFS are also given in table 1.

Table 1: Input data and Results (IBFS)

Ex.	Input Data	Obtained allocations by applying S. K. Goyal's approach	TC	Obtained allocation by applying TOM	TC
1	$[c_{ij}]_{4\times5}$=[10 8 4 5 13; 7 9 8 10 4; 9 3 7 10 6; 11 4 8 3 9]; $[a_i]_{4\times1}$=[100, 80, 70, 90]; $[b_j]_{1\times5}$=[60, 40, 100, 50, 90]	x_{11} = 20(5), x_{13} = 80(6), x_{25} = 80(3), x_{32} = 40(1), x_{33} = 20(5), x_{34} = 10(4) ,x_{41}= 40(8), x_{44}=50(2)	2170	x_{11} = 60(7), x_{13} = 30(6), x_{15} = 15(8), x_{25} = 80(3), x_{32} = 40(1), x_{33} = 30(4) ,x_{43}= 40(5), x_{44}= 50(2)	2120

So total transportation cost is increased by using S. K. Goyals approach. Because in Kirca and Satirs approach there was a tie for making 6th allocation i. e. in selecting the cell (1,3) and cell (4,3). Using tie breakers the cell (4,3) was selected because it had corresponding minimum unit transportation cost. But according to S. K. Goyal, the cell (1,3) had less opportunity cost than cell (4,3). So allocation was first given to cell (1,3) but it had corresponding unit transportation cost is more. After giving allocation to that cell remaining allocations were given to the cells with corresponding maximum unit transportation cost. So for this reason unit transportation cost is increased.

Case 2

In many cases corresponding minimum opportunity cost cells in both approaches are same, therefore using least cost method both approaches yields same solutions. Counter example to explain case 2 is given in table 2 and results obtained by applying both existing techniques to find IBFS are also given in table 2.

Table 2: Input data and Results (IBFS)

Ex.	Input Data	Obtained allocations by applying S. K. Goyal's Approach	TC	Obtained allocation by applying TOM	TC
2	$[c_{ij}]_{3\times3}$=[4 3 5; 6 5 4; 8 10 7]; $[a_i]_{3\times1}$=[90, 80, 100]; $[b_j]_{1\times3}$=[70, 120, 80]	x_{12} = 90(1), x_{23} = 80(2), x_{31} = 70(3), x_{32} = 30(4	1450	x_{12} = 90(1), x_{23} = 80(2), x_{31} = 70(3), x_{32} = 30(4)	1450

Solution using both approaches is same because corresponding cells with minimum opportunity cost are same. Therefore allocations are made in same cells in both tables because of same method.
So in both above cases S. K. Goyal's [4] improvement in TOM is not much helpful in obtained better initial basic feasible solution than TOM, if method is adopted for making allocations. In this paper, we have applied different method for making allocations in the opportunity cost matrix obtained by S. K. Goyal's [4] which gives most of times better initial basic feasible solution to transportation problems (TP).

2.1. Proposed Modified Approach

The following steps are involved in our proposed heuristic and see reference [7] for notations:

Step1.
Identify the smallest unit transportation cost C_{ij} from each row of transportation problem and calculate row opportunity cost E_{ij} by subtracting it from each C_{ij} of the corresponding row.

Step2.
Indentify the smallest unit transportation cost C_{ij} from each column of transportation problem and calculate Column opportunity cost F_{ij} by subtracting it from each C_{ij} of corresponding column.

Step3.
Choose opportunity cost T_{ij} = max (E_{ij}, F_{ij})

Step4.
Find R_i = largest T_{ij} - smallest T_{ij} , for all $j = 1, 2, 3, \ldots m$ and fixed i

Step5.
Find K_j = largest T_{ij} - smallest T_{ij} ,for all $i = 1, 2, 3, \ldots n$ and fixed j

Step6.
Choose max (R_i, K_j) and make the maximum possible allocation in cell which has smallest cost in corresponding row/column to that maximum penality.If there is a tie in selecting the maximum penalty.then use following tie breakers:

6a. Choose the row or column which has smallest T_{ij}

6b. If again tie is exists in (a) then choose the row or column which has corresponding smallest C_{ij}

6c. If again tie exists in (b) then avoid the cell which leads to degeneracy.

Step7.
Cross the row or column for which supply or demand is satisfied and repeat above procedure for making allocations in remaining rows and columns

2.2. Application of Proposed modified Approach

Examples given in table 1 and table 2 are also solved using proposed modified approach. Results are given in table 4.

Table 3: Results (IBFS) of examples given in case 1 and case 2

Ex.	Obtained allocations by applying our proposed modified Approach	TC
1	x_{11} = 60(7), x_{13} = 40(6), x_{25} = 80(1), x_{33} = 60(5), x_{35} = 10(4) ,x_{42}= 40(3), x_{44}= 50(2)	2070
2	x_{12} = 90(1), x_{22} = 30(2), x_{23} = 50(4), x_{31} = 70(3) , x_{33}=30(5)	1390

3. Note on JHM method

From existing methods, recentely developed JHM method is given by Juman et al. [5]. In which, first they allocate respective demand quantities in minimum cost cell of corresponding column and then adjust excess supply to next minimum cost cells by using some rules. But they have not given any rule for breaking tie occurs in selecting minimum cost cell for assigning respective demand quantities. Because when we select minimum cost cell arbitrarily for assigning respective demand quanity, obtained total transportation cost is different on selecting different minimum cost cells. So we can not assure that, the minimum cost cell which we are selecting arbitrarily, will give efficient solution or not.
In this note, we have made completion in JHM method by using rule for breaking ties in selecting least cost cell, which s gives better initial feasible solution of transportation problem.

3.1. Complete JHM Algorithm

Step 1: Express the given transportation problem into the form of m matrix with transportation cost c_{ij}, supply quantities a_i and demand quantities b_j for $i = 1,2,...m$ and $j = 1,2,...n$.

Step 2: Select the minimum cost cell from each column and assign respective demand quantities (b_j, $j = 1,2,...n$). If tie occurs in selecting minimum cost cell then select that minimum cost cell, which has corresponding maximum supply.

Step 3: Cross out the rows for which respective supply quantity is satisfied and go to step 9 if sum of assigned allocations for uncrossed rows is less than or equal to respective supply quantities.

Step 4: For each allocation of unmet rows with excess supply quantity, obtain the difference between second least and least unit cost of the columns contained that allocation and choose the allocation with smallest difference (in case of tie, choose the one with largest unit cost) . If there exists only one unmet row then go to step 9, otherwise choose the smallest difference for each unmet row separately and go to step 5.

Step 5: If there exist an unmet row, which does not contains second least unit cost corresponding to the smallest difference between second least and least unit costs of column for each of the allocation of another unmet row then identify that unmet row and go to step 7.

Step 6: Choose any two unmet rows and calculate the differences between second least and least unit costs of columns containing allocations in both unmet rows. Let smallest of the differences for both unmet rows be corresponding to the least cost g_1 and f_1 and let g_1, g_2, g_3 and f_1, f_2, f_3 be 1^{st}, 2^{nd} and 3^{rd} least unit costs in two columns and if $(f_3 - f_1 > g_3 - g_2)$, then identify the unmet row contained the least unit cost f_1. otherwise, identify the other unmet row.

Step 7: Transfer maximum possible amount of the excess supply quantity from the least unit cost cell to the second least unit cost cell in column of identified unmet row in step 4 (or step 5 or step 6) corresponding to the smallest of the differences between second least and least unit costs of column for each of allocation in this row. If excess supply remain there, then transfer excess supply quantity by using same procedure for next smallest difference between second least and least unit costs and continue till whole excess supply transferred from identified unmet row.

Step 8: Cross out the row for which excess supply quantity is completely removed and go to step 3.

Step 9: Stop and consider the current solution as IBFS.

3.2. Application

we have used numerical examples given by Jumen et al. [5] as test problems. Input data and source of numerical examples is given in table 4

Table 4: Input data and Source of examples

Ex.	Input Data	Source
1	$[c_{ij}]_{3\times4}$=[4 6 8 8; 6 8 6 7; 5 7 6 8]; $[a_i]_{3\times1}$=[40, 60, 50]; $[b_j]_{1\times4}$=[20, 30, 50, 50]	Juman et al.[5]
2	$[c_{ij}]_{4\times6}$=[9 12 9 6 9 10; 7 3 7 7 5 5; 6 5 9 11 3 11; 6 8 11 2 2 10]; $[a_i]_{4\times1}$=[5, 6, 2, 9]; $[b_j]_{1\times6}$=[4, 4, 6, 2, 4, 2]	Juman et al. [5]

In example 1, tie occurs in selecting minimum cost cell c_{23} and c_{33} (because, c_{23}=c_{33}) for assigning demand quantity b_3. According to JHM, if we assign b_3 in the cell c_{23}, then by applying JHM, we found total transportation cost as 920 and if we assign b_3 in the cell c_{33}, then total transportation cost is obtained as 930. So Results obtained by applying JHM method are given in table 5.

Table 5: Results (IBFS) of example 1

Ex.	Obtained Allocations (on selecting cell c_{23})	Obtained Total Cost	Obtained Allocations (on selecting cell c_{33})	Obtained Total Cost
1	x_{11} = 20, x_{12} = 20, x_{23} = 10, x_{24} = 50, x_{32} = 10, x_{33} = 40	920	x_{11} = 20, x_{12} = 20, x_{22} = 10, x_{24} = 50, x_{33} = 50	930

Similarly in example 2, there is a tie in selecting minimum cost cells c_{31} and c_{41} for assigning demand quantity b_1. By applying JHM, total transportation costs are obtained as 114 and 112 by assigning demand quantity b_1 to c_{31} and c_{41} respectively. Obtained results by JHM method of example 2 are given in table6.

Table 6: Results of example 2

Ex.	Obtained Allocations (on selecting cell c_{31})	Obtained Total Cost	Obtained Allocations (on selecting cell c_{41})	Obtained Total Cost
2	x_{13} = 5, x_{22} = 4, x_{26} = 2, x_{31} = 2, x_{41} = 2, x_{43} = 1 ,x_{44} = 2, x_{45} = 4	114	x_{13} = 5, x_{22} = 4, x_{26} = 2, x_{31} = 1, x_{33} = 1,x_{41} = 3, x_{44} = 2,x_{45} = 4	112

Solution of example 1 and example 2 using complete JHM method is given in table 7.

Table 7: Results (IBFS) of examples

Ex.	Obtained Allocations	Obtained Transportation cost
1	$x_{11} = 20, x_{12} = 20, x_{23} = 10, x_{24} = 50, x_{32} = 10, x_{33} = 40$	920
2	$x_{13} = 5, x_{22} = 4, x_{26} = 2, x_{31} = 1, x_{33} = 1, x_{41} = 3, x_{44} = 2, x_{45} = 4$	112

So solution obtained by complete JHM is best initial solution and after being checked by MODI method [2], the above initial solution can be shown as optimal solution.

4. Conclusions

From above theory and application , it is concluded that when further improvement is made in S. K. Goyal's modified TOM and JHM method, it gives better initial basic feasible solution, which is nearer to optimal solution or some times reach optimal solution.

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