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Common Weight Decompositions of Some Snake Related Graphs

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ABSTRACT: *A difference labeling of a graph G is realized by assigning distinct integer values to its vertices and then associating with each edge the absolute difference of those values assigned to its end vertices. A decomposition of labeled graph into parts, each part containing the edge having a common- weight is called a common – weight decomposition. In this paper investigate the existence of triangular snake, double triangular snake, quadrilateral snake, double quadrilateral snake, alternative quadrilateral snake.*

KEYWORDS: *Difference labeling, common weight decomposition.*

1. INTRODUCTION

In this paper, we consider only finite simple undirected graph. In which G has vertex set *V=V(G)* and edge set *E=E(G)* the set of vertices adjacent to vertex *u* of *G* is denoted by *N= N(u).* For the notation and terminology we referred to Bondy and Murthy [2].

A difference labeling of a graph G is realized by assigning distinct integer values to its vertices and then associating with each edge the absolute difference uv of those values assigned to its end vertices. The concept of difference labelings was introduced by Bloom and Ruiz [1] and was further investigated by Arumugam and Meena [6]. Meena and Vaithilingam [7] have investigated the existence difference labelings whereas crown graph, grid graph, pyramid graph, fire cracker, banana trees, gear graph, ladder, fan graph, friendship graph, helm graph, wheel graph and $P_{2n}(+)N_{m}$. In addition, various labelings graphs problem have been examined by Jeyanthi and Saratha Devi [8]; and Manisha [9].

Definition: 1.1. Let G= (V,E) be a graph. A difference labeling of G is an injection from V to the set of non-negative integers with weight function f* on E given $f*(uv) = |f(u)-f(v)|$ for every u edge in G. A graph with a difference labeling defined on it is called a labeled graph.

Definition: 1.2. A decomposition of labeled graph into parts, each part containing the edge having a common- weight is called a common – weight decomposition.

Definition: 1.3. A common weight decomposition of G in which each part contains m edges is called m-equitable.

Definition: 1.4. Specified Parts Decomposition Problem is a given graph G with edge set E(G) and a collection of edge – disjoint linear forests $F_1, F, F_3, ..., F_k$ containing a total of $|E|$ edges , does there exists a common weight decomposition of G whose parts are respectively isomorphic to $F_1, F_2, F_3, ..., F_k$.

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Definition 1.5 : An alternative Quadrilateral Snake A(Q S_n) is obtained from a path u_1, u_2….,u_n by joining u_iu_{i+1} (alternatively) to new vertices v_i and v_{i+1} and then joining v_iv_{i+1}. That is every alternative edge of a path is replaced by a cycle $C_{4,r.}$

Definition 1.6: Triangular snake T_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le n-1$, that is every edge of path is replaced by a triangle C_3.

Definition 1.7: A **double triangular snake** $D(T_n)$ consists of two triangular snake that have a common path. which is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to a new vertex $v_i \left(1 \le i \le n-1\right)$ and to a new vertex $w_i \left(1 \le i \le n-1\right)$.

Definition 1.8: A **quadrilateral snake** Q_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to two vertices v_i and w_i, $1 \le i \le n-1$ respectively and then joining v_i and w_i.

Definition 1.9: A **double quadrilateral snake** DQ_n consists of two triangular snakes that have a common path. Let

$V\left(DQ_n\right) = \left\{u_i \ /1 \le i \le n\right\} \cup \left\{v_i, w_i, x_i, y_i \ 1 \le i \le n-1\right\}$ and

$E\left(DQ_n\right) = \{u_iu_{i+1}, v_iw_i, x_iy_{i+1}, y_iu_{i+1} / 1 \le i \le n-1\}$

MAIN RESULTS

Theorem 2.1:

There exists a labeling which realizes a common weight decomposition of the alternate triangular snake $A(T_n)$ for n is even, n≥ 2 into a copy of $\left(\frac{n}{2}\right) P_2$ and a copy of $P_{\frac{3n}{2}}$

Proof:

Let G be a alternate triangular $A(T_n)$ graph which is obtained from the path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to new vertex $V_{\frac{i+1}{2}}$ for i= 1, 3,..., n – 1.

Define a vertex labeling

$$f : V\left(G\right) \to \left\{0,1,...,n\right\} \text{ as follows}$$

$$f\left(u_i\right) = \frac{3i}{2} - 1 \qquad \text{for } i = 2, 4, 6,..., n$$

$$f\left(u_i\right) = \frac{3\left(i+1\right)}{2} - 3 \qquad \text{for } i = 1, 3, 5,..., n-1$$

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$$f(v_i) = 3i - 2 \qquad \text{for } 1 \le i \le \frac{n}{2}$$

Then the set of edges S_1 and S_2 forms a common weight decomposition of alternate triangular snake $A(T_n)$ for n is even into a copy of $\left(\frac{n}{2}\right) P_2$ and a copy of $P_{\frac{3n}{2}}$. With weights 2,1 respectively.

Where

$$S_1 = |\left\{u_i u_{i+1} \; i = 1,3,5,\ldots,n-1\right\}$$

$$S_2 = \left\{u_1 v_1 u_2 u_3 v_2 \ldots u_{n-1} v_{\frac{n}{2}} \; u_n\right\}.$$

Theorem 2.2:

There exists a labeling which realizes a common weight decomposition of $A(T_n)$ for n is odd into a copy of $\left(\frac{n-1}{2}\right) P_2$ and a copy of $P_{\frac{3n-1}{2}}$.

Proof:

Let G be a alternate triangular graph $A(T_n)$ which is obtained from the path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to new vertex $V_{\frac{i+1}{2}}$ for i= 1, 3,..., n – 2

Define a vertex labeling $f : V(G) \to \{0,1,2\ldots,n\}$ as follows

$$f(u_i) = \frac{3i}{2} - 1 \qquad \text{for } i = 2, 4, \ldots, n-1$$

$$f(u_i) = \frac{3(i+1)}{2} - 3 \qquad \text{for } i = 1, 3, 5, \ldots, n$$

$$f(v_i) = 3j - 2 \qquad \text{for } 1 \le i \le \frac{n-1}{2}$$

Then the set of edges S_1 and S_2 forms a common weight decomposition of $A(T_n)$ for n odd into a copy of $\left(\frac{n-1}{2}\right) P_2$ and a copy of $P_{\frac{3n-1}{2}}$ with weights 2,1 respectively.

Where

$$S_1 = \left\{u_i u_{i+1} \mid i = 1,3,5,\ldots,n-2\right\}$$

$$S_2 = \left\{u_1 v_1 u_2 u_3 v_2 \ldots u_{n-2} v_{\frac{n-1}{2}} u_{n-1} u_n\right\}$$

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Theorem 2.3:

There exists a labeling which realizes a common weight decomposition of the double triangular snake graph $D(T_n)$ into two copies of $(n-2)\,P_3 \cup 2P_2$ and a Copy of P_n.

Proof:

Let $G = D(T_n)$ be the graph with vertex set

$V(G) = \{u_i / 1 \le i \le n\} \cup \{v_i\ w_i / 1 \le i \le n-1\}$ and the edge set

$E(G) = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i, u_i w_i / 1 \le i \le n-1\} \cup \{u_{i+1} v_i, u_{i+1} w_i / 1 \le i \le n-1\}$

Define a vertex labeling $f : V(G) \to \{0, 1, 2, \ldots, 2n+2\}$ as follows

$f(u_i) = 3i - 3$ for $1 \le i \le n$

$f(v_i) = 3i - 2$ for $1 \le i \le n-1$

$f(w_i) = 3i - 1$ for $1 \le i \le n-1$

Then the set of edges $S1$, $S2$ and $S3$ form a common weight decomposition of double triangular snake $D\,(T_n)$ into two copies of $(n-2)\,P_3 \cup 2P_2$ and a copy of Pn with weights 3,2,1 respectively.

Where

$S_1 = \{u_1 u_2 \ldots u_n\}$

$S_2 = \{v_i u_{i+1} w_{i+1} / 1 \le i \le n-2\} \cup \{u_1 w_1, v_{n-1} u_n\}$

$S_3 = \{w_i u_{i+1} v_{i+1} / 1 \le i \le n-2\} \cup \{u_1 v_1, w_{n-1} u_n\}$

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Theorem 2.4:

There exists a labeling which realizes a common weight decomposition of the Alternative Quadrilateral Snake $A(QS_n)$ for $n \geq 2$ and n is even into a perfect matching and copy of $P_n \cup \left(\frac{n}{2}\right) P_2$.

Proof:

Let $G = A(QS_n)$ be the graph with vertex set $V(G) = \{u_1, u_2 ..., u_n, v_1, v_2 ..., v_n\}$

and the edge set E(G) = $\{u\, u_{i+1}\ 1 \leq i \leq n\text{-}1\} \cup \{v_{2i-1}\, v_{2i} / 1 \leq i \leq \frac{n}{2}\} \cup \{(u,v)\ 1 \leq i \leq n\}$.

Define a vertex labeling f: $V(G) \longrightarrow \{0, 1, \ldots, 2n - 1\}$ as follows

$f(u_i) = i - 1,$ for $1 \leq i \leq n$

$f(v_i) = i - 1 + n,$ for $1 \leq i \leq n$

Then the set of edges S_1 and S_2 forms a common weight decomposition of Alternative Quadrilateral Snake $A(QS_n)$ for $n \geq 2$ and n is even into a perfect matching a copy of $P_n \cup \left(\frac{n}{2}\right) P_2$.

Where

$$S_1 = \{u_i v_i\ 1 \leq i \leq n\}$$

$$S_2 = \{u_1, u_2 \ldots\ldots u_n\} \cup \{v_{2i-1} \cup V_{2i} / 1 \leq i \leq \frac{n}{2}\}$$

Theorem 2.5:

There exists a labeling which realizes a common weight decomposition of the alternative quadrilateral snake $A(QS_n)$ for n is even, $n \geq 2$ into two perfect matchings and a copy of $\left(\frac{n}{2} - 1\right) P_2$.

Proof:

Let $G = A(QS_n)$ be the graph with vertex set $V(G) = \{u_1, u_2 ..., u_n, v_1, v_2 ..., v_n\}$ and the edge set

E(G) = $\{u_i\, u_{i+1}\ 1 \leq i \leq n\text{-}1\} \cup \{v_{2i-1} \cup V_{2i} / 1 \leq i \leq \frac{n}{2}\}\ \{(u_i\, v_i)\ 1 \leq i \leq n\}$

Define a vertex labeling $f : V(G) \rightarrow \{0, 1, ..., 2n - 1\}$ as follows

$$f(u_1) = 0$$

$$f(u_i) = i, \text{ if i is odd, } i = 2m + 1 \text{ when m is odd, } 3 \leq i \leq n - 1$$

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$f(u_i) = i-1$ if i is odd , $5 \le i \le n-1$, $i = 2m+1$ where m is even

$f(u_i) = i-1$, if i is even, $2 \le i \le n$ $i = 2m$ where m is odd

$f(u_i) = i-2$ if i is even, $4 \le i \le n$ $i = 2m$ where m is even

$f(v_i) = f(u_i) + n$, $1 \le i \le n$

Then the set of edges S1,S2,S3 forms a common weight decomposition of alternative quadrilateral snake A(QS_n) into two perfect matchings and copy of $\left(\frac{n}{2}-1\right)P_2$.

Where

$S_1 = \{u_i v_i \,/ 1 \le i \le n\}$

$S_2 = \{u_i u_{i+1}, v_i v_{i+1} / 1 \le i \le n-1,\ i \text{ is odd}\}$

$S_3 = \{u_i\, u_{i+1} / \ 2 \le i \le n-2,\ i \text{ is even}\}$.

Theorem 2.6:

There exists a labeling which realizes a common weight decomposition of $T_n AK_1$ graph decomposed into a copy of P_n, a copy of $(n-1)\ P_4 \cup P_2$ and a copy of $(n-1)\ P_2$.

Proof:

Let $G = T_n AK_1$ be the graph with vertex set $V(G) = \{x_i, u_i 1 \le i \le n\} \cup \{v_i, y_i 1 \le i \le n-1\}$ and the edge set $E(G) = \{u_i u_{i+1}\ 1 \le i \le n-1\} \cup \{u_{i+1} v_i 1 \le i \le n-1\} \cup \{u_i v_i, u_i y_i 1 \le i \le n-1\} \cup \{u_i x_i\ 1 \le i \le n\}$

Here $|V(G)| = 4n-3$ and $|E(G)| = 4n-2$.

Define a vertex labeling $f : V(G) \to \{0, 1, 2 \ldots 4n-3\}$ as follows

$f(x_i) = 4(i-1)$ for $1 \le i \le n$

$f(u_i) = 4i-3$ for $1 \le i \le n$

$f(v_i) = 4i-2$ for $1 \le i \le n-1$

$f(y_i) = 4i-1$ for $1 \le i \le n-1$

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Then the set of edges S_1, S_2 and S_3 forms a common weight decomposition of T_nAK_1 graph decomposed into a copy of P_n, a copy of $(n-1)P_2$ and a copy of $(n-1)\ P_4 \cup P_2$ with weight 4, 3,1 respectively,

where

$$S_3 = \{u_1u_{i+1}/1 \le i \le n-1\}$$

$$S_2 = \{u_{i+1}v_i/1 \le i \le n-1\}$$

$$S_3 = \{x_iu_iv_i\,y_i\ \ 1 \le i \le n-1\} \cup \{u_nx_n\}$$

Conclusion

In this paper we investigate the existence of difference labellings for some snake related graphs. The Specified parts decomposition problem can be investigated for other classes of graphs.

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