COMPARATIVE THERMAL BUCKLING ANALYSIS BETWEEN ALUMINUM AND FGM RECTANGULAR PLATE

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ABSTRACT

This paper presents the results of a thermal buckling analysis for clamped rectangular plates. The analysis done on the two different materials i.e. aluminum and functionally graded material (FGM). In FGM rectangular plate power law considered for the gradation and gradation is done across the thickness. Comparison between the aluminum plate and FGM plate shows good agreement. This whole study done using the FEA software package COMSOL Multiphysics version 5.3.

Keywords: FGM plate; Buckling; thermal buckling; uniform temperature profile; COMSOL Multiphysics.

I. INTRODUCTION

The use of advanced composite materials in aerospace, automotive and marine technologies, etc. has been growing rapidly due to its excellent directional stiffness properties. Because of the advancement in the field of advanced composite materials of structural members have stimulated interest in the accurate prediction of the response characteristics of functionally graded (FG) plates used in situation where large temperature gradients encountered. Functionally graded materials (FGMs) are designed so that material properties vary smoothly and continuously through the thickness surface of a ceramic exposed to high temperature to that of a metal on the other surface. The mechanical properties graded in the thickness according to volume fraction power law distribution. The thermal buckling analysis of FGM plate has been investigated using the finite element method by Na and Kim[2] [3] [4] employed 18-node solid element and assumed the strain mixed formulation for 3D thermalbuckling and post-buckling analysis of FGM plates with temperature dependent properties. Jaberzadeh et. al.[5] used EFG method and approximation to study the thermal buckling of FGM skew and trapezoidal plates based on CPT. Jalali et.

al.[6] presented the finite element method, which involves pseudo-spectral method, and collocation method to examine the thermal stability of laminated FGM circular plates based on FSDT and subjected to uniform temperature rise. Zandekarimi[7] investigated the size-dependent thermalbuckling and post-buckling behaviour of FG circular micro-plate under uniform temperature rise field and in clamped boundary conditions.

Lanhe et. al.[8] practiced the dynamic stability of thick FGM plates based on the FSDT and subjected to aerothermo- mechanical loads. Lee et. al.[9] extended the post-buckling analysis of FGM plates based on FSDT under edge compression temperature field condition. Malekzadeh[10] presented 3D thermal buckling analysis using differential quadrature technique, and obtained results were validated by comparing them with HSDT. Ghannadpour[11] used the finite strip method for buckling analysis of FGM plates under thermal loading based on CPT and three types of loading were considered i.e. uniform temperature rise, linear temperature change across the thickness and nonlinear temperature change across the thickness. Zhang et. al.[12] analysed the mechanical and thermal buckling behavior of FGM plates based on FSDT using MLPG approach along with moving Kriging interpolation technique, which has Kronecker delta function property. Tran[13] presented study intends to analyze nonlinear buckling behavior of functionally graded (FG) plates under thermal loading by a mesh-free method. Buckling formulations is derived by HSDT.

This paper present comparative thermal buckling study between the aluminium plate which have isotropic property and FGM plate with the power law function. The whole study done using FEA sftware package COMSOL Multiphysics version 5.3.

II. FUNDAMENTAL EQUATIONS OF FUNCTIONALLY GRADED PLATES

The methods for evaluating the effective material properties across the plate's thickness are discussed below.

1) Power Law Function (P-FGM)

The methods of estimation are based on linear rule of mixture and has been extensively used in the literature to investigate the response of FGM [14] to [15].

The properties of material of P-FGM can explained by the rule of mixture:

$$\boldsymbol{P(z) = (P_t - P_b)V_f + P_b} \qquad \mathbf{1}$$

Properties of material are dependent on the volume fraction V_f of P-FGM that obeys power law,

$$\boldsymbol{V_f = \left(\frac{z}{h} + \frac{1}{2}\right)^n} \qquad \mathbf{2}$$

where n is a parameter that symbolize the material variation profile through the thickness known, as is the volume fraction exponent. A bottom face, (z/h) = -1/2 and $V_f = 0$, hence P(z) = Pb and at top face, (z/h) = ½ and so $V_f = 1$, hence $P(z) = P_t$ where P denotes a generic material property like modulus, P_t and

Pb denotes the property of the top and bottom faces of the plate. At n = 0 the plate is a fully ceramic plate while at n = ∞ the plate is fully metal.

2) Sigmoid Law Function (S-FGM)

In the case of adding an FGM of a single power-law function to the multi-layered composites, stress concentrations appear on one of the interfaces where the material is continuous but changes rapidly. The volume fraction using two power-law functions to ensure smooth distribution of stresses among all the interfaces. The two power-law functions defined by:

$$G_1 = 1 - \frac{1}{2}\left(\frac{\frac{h}{2} - z}{\frac{h}{2}}\right) p \; for \; 0 \le z \le \frac{h}{2} \; and$$

$$G_2 = \frac{1}{2}\left(\frac{\frac{h}{2}+z}{\frac{h}{2}}\right) p \; for \; \frac{-h}{2} \le z \le 0 \qquad 3$$

These combinations of two-power law functioncalled sigmoid law.

3) Exponential Law Function (E-FGM)

The material distribution of E-FGM plate shown by equation,

$$E(z) = E_2 e^{\frac{1}{h} ln\left(\frac{E_1}{E_2}\right)\left(z+\frac{h}{2}\right)}$$

III. BUCKLING ANALYSIS

In this section, a model developed to predict the temperature at which the flat equilibrium configuration of a rectangular plate loses stability. The numerical approach presented here requires that the in-plane boundary conditions are fully fixed (no in-plane displacements at the edges) while a variety of out-of-plane conditions maybe considered. For the sake of the analysis, the results of this model will focus on plates with fully clamped out-of-plane boundary conditions on all four sides. Because initial buckling corresponds to an unbounded solution in the linearized system, a linear analysis is sufficient to obtain the thermal buckling characteristics of the plate. In dimensional form, the linearized strain energy per unit thickness, U, for the flat plate takes the form

$$U = \int_0^a \int_0^b [N_x \epsilon_{xs} + N_y \epsilon_{ys} + N_{xy} \epsilon_{xys}] dxdy + \frac{D}{2} \int_0^a \int_0^b [W_{xx}^2 + W_{yy}^2 + 2\vartheta W_{xx} W_{yy} + 2(1 - \vartheta) W_{xy}^2] dxdy \qquad 4$$

where W is the out-of-plane displacement field, D is the bending stiffness, N_x and N_y are the in-plane loads, $\square_{xs}$, $\square_{ys}$, $\square_{xys}$ are the stretching components of the strains, and ν is Poisson's ratio. Note that the first term represents the energy due to in-plane stretching while the second term corresponds to the bending energy. In addition, the compressive in-plane biaxial loads are assumed proportional to the temperature rise (Boley and Weiner, 1965).

IV. STUDY OF ALUMINUM RECTANGULAR PLATE

Aluminum is the isotropic material used for the thermal buckling analysis. There are various applications of aluminum in aerospace industry due to low density. Therefore, aluminum plate considered for the thermal buckling analysis.

Mechanical properties of Aluminum are as follows:

Material	Young's Modulus, E (GPa)	Poisson's Ratio, (ν)	Coefficient of thermal expansion, α (/K)	Thermal conductivity, K $\left(\frac{W}{mK}\right)$
Aluminum	70	0.30	23×10^{-6}	204

The contours plots are shown below for the clamped (CCCC) aluminum plate for the thermal buckling analysis.

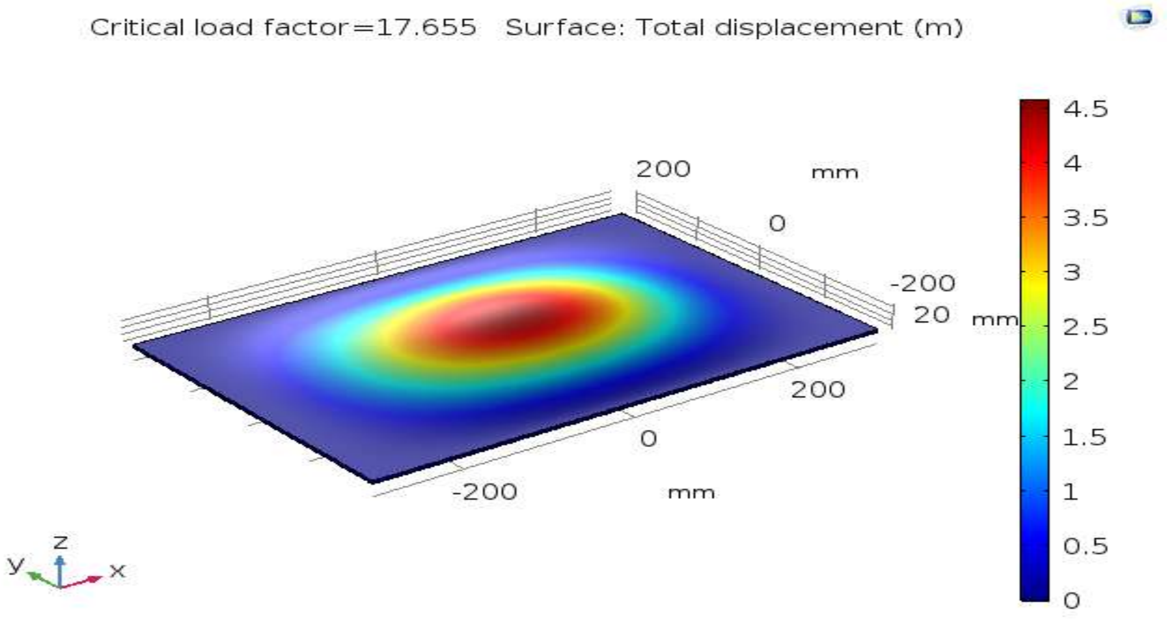

Figure 1: Critical buckling temperature (ΔT_{cr}) of Aluminum plate.

V. STUDY OF RECTANGULAR FGM PLATE

Mechanical properties of FGM rectangular plate.

Material	Young's Modulus, E (GPa)	Poisson's Ratio, ν	Coefficient of thermal expansion, α (/K)	Thermal Conductivity, K $\left(\frac{W}{m}\right.$
Aluminum (Metal)	70	0.30	23×10^{-6}	204
Alumina (Ceramic)	380	0.30	7.4×10^{-6}	10.4

In this study bottom surface of plate is pure metal i.e. aluminum and top surface is ceramic i.e. alumina (Al_2O_3). The volume fraction index is considered as n = 0, 0.5, 1, 5, 10. The contours of following volume fraction index are shown below.

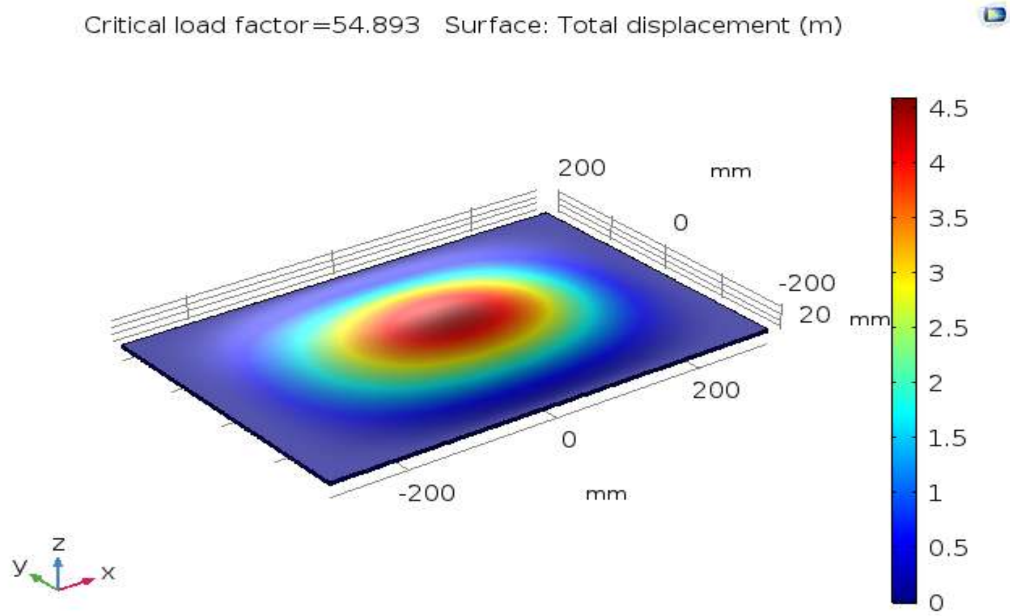

Figure 2: Critical buckling temperature (ΔT_{cr}) of FGM rectangular plate for n = 0.

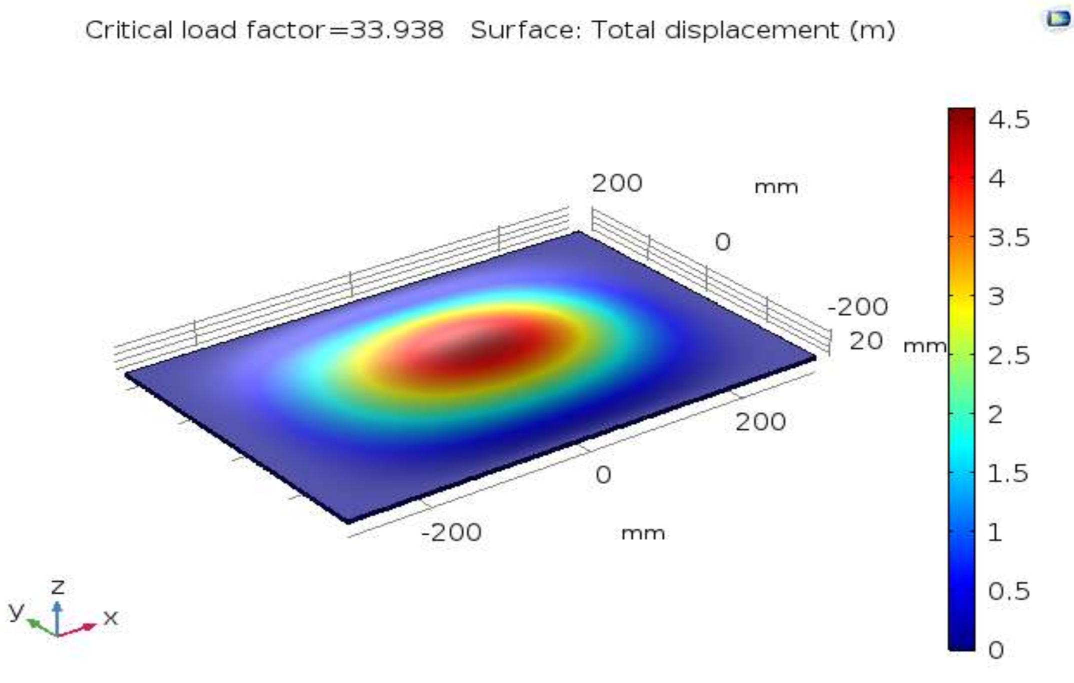

Figure 3: **Critical buckling temperature (ΔT_{cr}) of FGM rectangular plate for n = 0.5.**

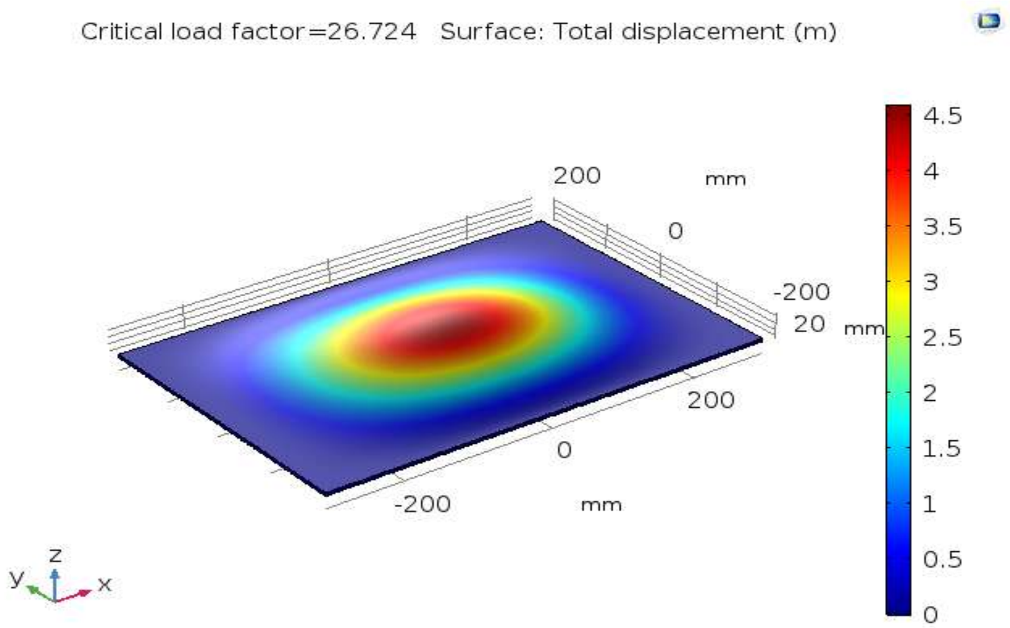

Figure 4: **Critical buckling temperature (ΔT_{cr}) of FGM rectangular plate for n = 1.**

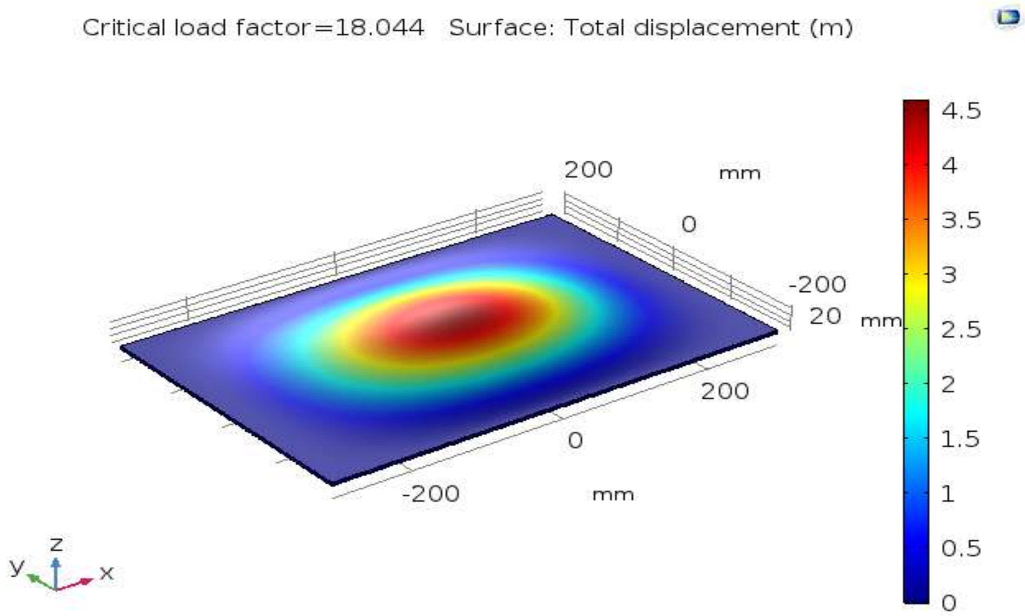

Figure 5:Critical buckling temperature (ΔT_{cr}) of FGM rectangular plate for n = 5.

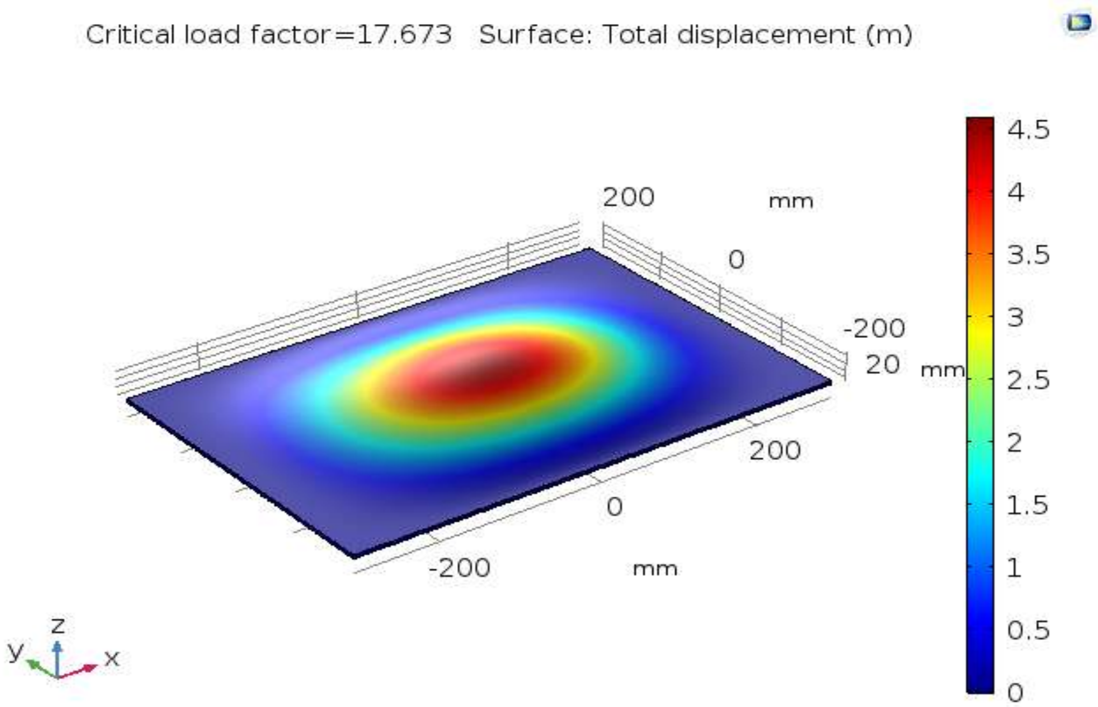

Figure 6: Critical buckling temperature (ΔT_{cr}) of FGM rectangular plate for n = 10.

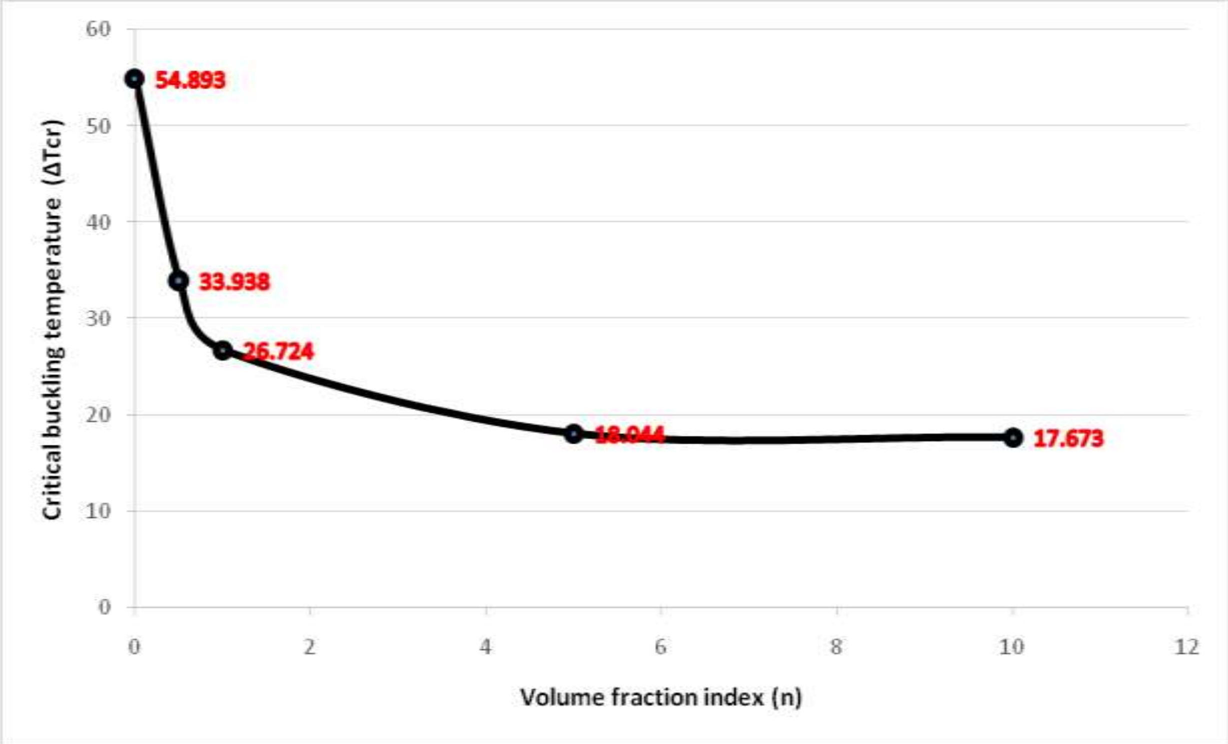

Figure 7: Graphical representation of critical buckling temperature (ΔT_{cr}) and volume fraction index (n).

In the thermal buckling analysis of FGM rectangular plate, analysis done by varying volume fraction index from n = 0 to n = 10. As it is very clear, that volume fraction index increases it show metal-rich properties in FGM. From this study the observation, shown in Figure 7 that as the volume fraction index (n) increases the critical buckling temperature (ΔT_{cr}) decreases.

VI. CONCLUSION

Critical buckling temperatures (ΔT_{cr}) of clamped rectangular FG plates and aluminum plate have been analyzed by using FEA-software package COMSOL Multiphysics using first-order shear deformation theory. The comparison of the aluminum plate and FGM plate shows following observations.

i. It is shown through the Figure 1 that the ΔT_{cr} of aluminum plate is low which suggest that the thermal buckling in aluminum plate is easily entertained.

ii. In the study of FGM plate it is observed from Figure 7 that when the value of n increases the ΔT_{cr} decreases.

iii. This study suggest that ceramic-rich FGM is stronger than the metal-rich FGM in case of thermal buckling analysis.

iv. In case of comparison of aluminum and FGM plate, ceramic-rich FGM is more thermal buckling resistant than aluminum plate.

REFERENCES

[1] Yadav Vishal, Rathore S.K., 2018. Buckling and Post-buckling behavior of FGM plate – A Review. International Journal of Advance Research in Science and Engineering, Vol. 07, ISSN: 2319-8354.

[2] K.J. Na KS, 2004. Three dimensional thermal buckling analysis of functionally graded materials. Composite Structures, Part B.

[3] K.J. Na KS, 2006. Thermal postbuckling investigations of functionally graded plates using 3-D finite element method.

[4] K.J. Na KS, 2006. Three-dimensional thermomechanical buckling analysis for functionally graded composite plates. Composite Structures.

[5] Jaberzadeh E., 2013. Thermal buckling of functionally graded skew and trapezoidal plates with different boundary conditions using the element-free Galerkin method.

[6] Jalali S.K., 2010. Thermal stability analysis of circular functionally graded sandwich plate of variable thickness using pseudo-spectral method.

[7] Zandekarimi R.S., 2017. Size dependent thermal buckling and postbuckling of functionally graded circular microplates based on modified couple stress theory, Journal of Thermal Stresses.

[8] Lanhe W, 2007. Dynamic stability analysis of FGM plates by the moving least square differential quadrature method, Composite Structures.

[9] Lee YY, 2010. Postbuckling analysis of functionally graded plates subject to compressive and thermal loads.

[10] Malekzadeh P, 2011. Three-dimensional thermal buckling analysis of functionally graded arbitrary straight-sided quadrilateral plates using differential quadrature method, Composite Structures.

[11] Ghannadpour SAM, 2012. Buckling analysis of functionally graded plates under thermal loadings using the finite strip method, Composite Structures.

[12] Zhang LW, 2014. Thermal buckling of functionally graded plates using Kriging meshless method. Composite Structures.

[13] Minh Tung Tran, 2017, Nonlinear thermal buckling analysis of functionally graded plates by a mesh-free radical point interpolation method, Engineering analysis with boundary elements.

[14] C. CC, "Simplified composite micromechanics equations for hygral, thermal and mechnical properties," 1983.

[15] C. C. Hopkins D.A., "A unique set of micromechanics equations for high temperature metal matrix composites.,"1985.

[16] A. J. Feldman E, "Buckling analysis of functionally graded plates subjected to uniaxial loading,"1997.

[17] W. T. Ma L.S., "Relationships between axisymmetric bending and buckling solutions of FGM circular plates based on third-order plate theory and classical plate theory,"2004.

[18] A. Zenkour, "A comprehensive analysis of functionally graded sandwich plates: Part 2 - Buckling and free vibration,"2005.

[19] A. M, "Conditions for functionally graded plates to remain flat under in-plane loads by classical plate theory," Composite Structures,2008.

[20] R. A. S. S. Saidi AR, "Axisymmetric bending and buckling analysis of thick functionally graded circular plates using unconstrained third-order shear deformation plate theory," Composite Structures,2009.

[21] S. A. J. E. Mohammadi M, "Levy solution for buckling analysis of functionally graded rectangular plates," Applied Composite Materials,2010.

[22] S. A. Naderi A, "On pre-buckling configuration of functionally graded Mindlin rectangular plates," 2010.

[23] T. A. Z. N. M. I. A. E. Meiche NE, "A new hyperbolic shear deformation theory for buckling and vibration of functionally graded sandwich plate," International Journal Of Mechanical Sciences,2011.

[24] C. D. Thai HT, "An efficient and simple refined theory for buckling analysis of functionally graded plates," Applied Mathematical Model,2012.

[25] S. RP, "Refined plate theory and its variants,"2002.

[26] F. F. K. M. Latifi M, "Buckling analysis of rectangular functionally graded plates under various edge conditions using Fourier series expansion,"2013.

[27] N. T. V. T. L. J. Thai HT, "Analysis of functionally graded sandwich plates using a new first-order shear deformation theory," 2014.

[28] H. M. T. A. M. S. B. O. Belabled Z, "An efficient and simple higher-order shear and normal deformation theory for functionally graded material (FGM) plates,"2014.

[29] B. E. A. Mohammed Sid Ahmed, A. Tounsi, M. Ahouel and Houari, "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept," Steel and Composite Structures, vol. 20, no. 5, pp. 963-981,2015.

[30] C. S. Pradhan KK, "Static analysis of functionally graded thin rectangular plates with various boundary supports.," 2015.