

A NOVEL APPLICATION OF FUZZY GRAPH IN IMAGE SEGMENTATION

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ABSTRACT

The goal of this paper is to produce a coloring algorithm based on fuzzy concept, so that a meaningful picture can be provided to decision makers to recognize possible fuzzy classes. A key argument, at least in land cover problems, is that our fuzzy classes have a core of connected pixels, surrounded by pixels showing some degree of membership to that core. Hence, classification is made pixel by pixel, but behavior of surrounding pixels should have a strong influence at every stage. This information was taken into consideration a later supervised analysis where additional information may also occur

Keywords: Graph, fuzzy graph, image processing, Welsh-Powell coloring problem, Indian sign language recognition.

1. INTRODUCTION

We know that a graph is a symmetric binary relation on a nonempty set V . Similarly, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The first definition of a fuzzy graph was by Kaufmann [1] in 1973, based on Zadeh's fuzzy relations [2]. But it was Azriel Rosenfeld [3] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. During the same time R.T. Yeh and S.Y. Bang [4.] have also introduced various connectedness concepts in fuzzy graphs. Fuzzy graph theory is now finding various applications in modern science and technology especially in the fields of information theory, neural network, expert systems, cluster analysis, medical diagnosis, control theory, pattern recognition etc. [5]. A fuzzy set is characterized by a membership function which assigns to each object a grade of membership and it ranges from 0 to 1.

Definition 1.1. A fuzzy graph is a pair of functions $G: (\sigma, \mu)$ where σ is a fuzzy subset X and μ is a symmetric relation on σ i.e. $\sigma: X \rightarrow [0, 1]$ and $\mu: X \times X \rightarrow [0, 1]$ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$

for all x, y in X . we denote the crisp graph of $G: (\sigma, \mu)$ by $G^*: (\sigma^*, \mu^*)$ where σ^* is referred to as the nonempty set X of nodes and $\mu^* = E \in X \times X$. Now crisp graph (X, E) is a special case of a fuzzy graph with each vertex and edge of (X, E) having degree of membership value 1 where loops are not consider and μ is reflexive. We Fuzzy graph coloring is one of the most important problems of fuzzy graph theory in which we partition the vertex (edge) set of any associated graph so that adjacent vertices (edges) belong to different sets of the partitions [6]. “Fuzzy Sets” has obtained the fuzzy analogues of several basic graph-theoretic concepts like bridges, paths, cycles, trees and connectedness and established some of their properties. Some connectivity concepts regarding fuzzy cut nodes and fuzzy bridges is also established. . A normal fuzzy graph is shown below in fig.1.

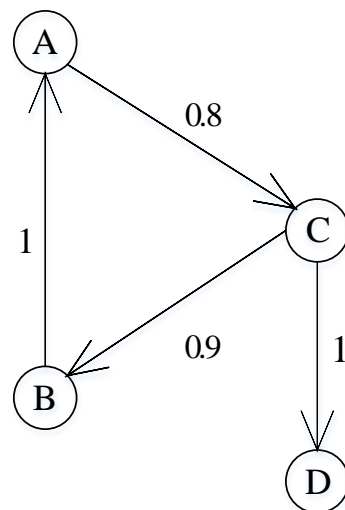


Figure 1: Fuzzy graph

.Fuzzy graph representation is more appropriate to reality than crisp graph representation. Every event in real world can be represented by fuzzy graphs appropriately. In this paper, a concept to color a fuzzy graph is discussed. This new coloring concept is used to color the sign images of hand gestures . A simple diagrammatic illustration of hand gestures for sign language is represented below in fig.2.



Figure 2: Hand gesture recognition

Sign language is used to express the thoughts of dumb peoples through hand gestures. Through the help of this language they can also communicate with others. Sign Language is a visual language, which consists of three major components such as finger-spelling- used to spell words letter by letter, word level sign vocabulary-used for the majority of communication and non-manual featuresfacial expressions and tongue, mouth and body position . Traditional segmentation of images is quite often assimilated to search for objects whenever clear borders exist, i.e., when sudden changes are observed when we move from one pixel to the next one. But natural entities may not show either a clear border or any standard shape, but a smooth gradation between different regions. In this paper, let us consider the image as fuzzy graph. Here, segmentation is done based on fuzzy graph .Here, fuzzy graph theory, is exploited for the segmentation process of ISL. A brief description of ISL recognition based on fuzzy graph is given below

3.1 ISL image representation using fuzzy graph

Let us assume an image as bi-dimensional map of pixels, each one of them being characterized by a fixed number of measurable attributes. These attributes can be the values of the three bands of the visible spectrum (red, green and blue). Therefore $P = (i, j)$ where $1 \leq i \leq r$, $1 \leq j \leq s$ will denote the set of pixel positions of an $r \times s$ image. If each pixel is characterized by b numerical measures, the whole image I can be characterized as $I = (x_{i,j}^1, \dots, x_{i,j}^b)$ where $(i, j) \in P$. For a given image I , the information of the b measures of any pixel $(x_{i,j}^1, \dots, x_{i,j}^b)$ can be represented by its position, $p = (i, j) \in P$ without confusion. Given such an image I , anyclassification problem follows a partition image regions, each being a subset of pixels, to be considered a candidate for a class. In this way, a welsh-powell classification approach

looks for a family of subsets of pixels $\{A_1, \dots, A_c\}$ such that $P = \bigcup_{k=1}^c A_k$ but $A_i \cap A_j = \emptyset \forall i \neq j$ where A_1, \dots, A_c is the family of classes explaining the image.

Fuzzy uncertainty appears when we consider a dissimilarity measure between pixels in order to identify possible homogeneous regions in the image. In any image classification problems, the selection of an adequate distance is a difficult issue that has been studied by many authors. Obviously, any classification process will be strongly dependent on the selection of the appropriate distance, to be chosen taking into account all features of the image under consideration, together with our particular classification objectives. Furthermore, in many instances the distance between two elements includes some lack of precision or ambiguity (it would be the case, for example, when we consider the aggregation of several measures obtained by different experts: a well-known problem in remote sensing is to choose an adequate distance in order to compare opinions from different experts). In order to capture the natural fuzzy uncertainty, and in order to give more flexibility to other already existing classification procedures, a fuzzy distance is considered which expresses the relation between the measured properties of pixels, $d : P \times P \rightarrow [0, \tilde{\infty})$ where $[0, \tilde{\infty})$ will be here the set of fuzzy numbers with domain in $R^+[10]$. We will denote by $d_{pp'} = d(p, p')$ the fuzzy distance between the pixels p and p' . We will denote its membership function by $\mu_{pp'} : R^+ \rightarrow [0, 1]$ and by $\tilde{D} = \{\tilde{d}_{pp'}; (p, p') \in P \times P\}$ we will denote its associated fuzzy distance matrix.

Given $r \times s$ image, a planar graph (P, E) can be defined considering P as the set of nodes and E the set of edges linking any couple of adjacent pixels. Two pixels $p(i, j), p'(i', j') \in P$ are adjacent if $|i - i'| + |j - j'| = 1$, i.e., if they share one coordinate being the other one contiguous. Let us now denote by $\tilde{G}(I) = (P, \tilde{E})$ the graph associated to the image I , where $\tilde{E} = \{\tilde{d}_{pp'}; (p, p') \text{adjacents}\}$ where $\tilde{d}_{pp'}$ are fuzzy numbers with domain in R^+ .



Figure 3:Representation of ISL

The size of the above image is 720×576 (*width* \times *height*) and the aspect ratio of this image size is 5×4 therefore the fuzzy graph representation of this image is given below in fig.4.

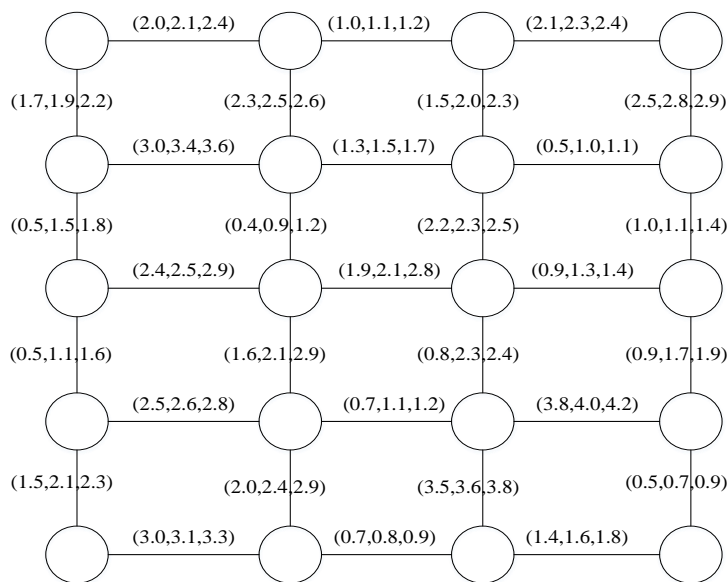


Figure 4: Pixels fuzzy graph

Definition 3.1: Given an image I and fuzzy distance d the pixels in fuzzy graph is represented by the pair $\tilde{G}(I) = (P, \tilde{E})$. Note that the pixels in fuzzy graph $\tilde{G}(I)$ can be defined by the set P and two $r \times s$ fuzzy matrices i.e. \tilde{D}^1 and \tilde{D}^2 . Subsequently the coloring procedure is based on the alternative representation, therefore the pixels in fuzzy graph $\tilde{G}(I)$ is denoted by $(r, s, \tilde{D}^1, \tilde{D}^2)$.

Example 3.1: Let $(r, s, \tilde{D}^1, \tilde{D}^2)$ be the pixels in fuzzy graph with $r = 5$, $s = 4$ and

$$\tilde{D}^1 = \begin{pmatrix} (1.7,1.9,2.2)(2.3,2.5,2.6)(1.5,2.0,2.3)(2.5,2.8,2.9) \\ (0.5,1.5,1.8)(0.4,0.9,1.2)(2.2,2.3,2.5)(1.0,1.1,1.4) \\ (0.5,1.1,1.6)(1.6,2.1,2.9)(0.8,2.3,2.4)(0.9,1.7,1.9) \\ (1.5,2.1,2.3)(2.0,2.4,2.9)(3.5,3.6,3.8)(0.5,0.7,0.9) \end{pmatrix}$$

$$\tilde{D}^2 = \begin{pmatrix} (2.0,2.1,2.4)(1.0,1.1,1.2)(2.1,2.3,2.4) \\ (3.0,3.4,3.6)(1.3,1.5,1.7)(0.5,1.0,1.1) \\ (2.4,2.5,2.9)(1.9,2.1,2.8)(0.9,1.3,1.4) \\ (2.5,2.6,2.8)(0.7,1.1,1.2)(3.8,4.0,4.2) \\ (3.0,3.1,3.3)(0.7,0.8,0.9)(1.4,1.6,1.8) \end{pmatrix}$$

Where each triple (a, b, c) represents a triangular fuzzy number. Such pixelsin fuzzy graph is represented in Fig. 6.

3.2 The basic Welsh-Powell coloring procedure

Welsh-Powell coloring algorithm

- Find the degree of each vertex.
- List the vertices in descending order.
- Color the first vertex in the list.
- Color every vertices in descending order one by one, such that the total number of colors used can be greater than or equal to the maximum degree.
- The overall complexity of this algorithm is $O(n^2)$.

A normal uncolored and undirected graph is shown below in fig.5.

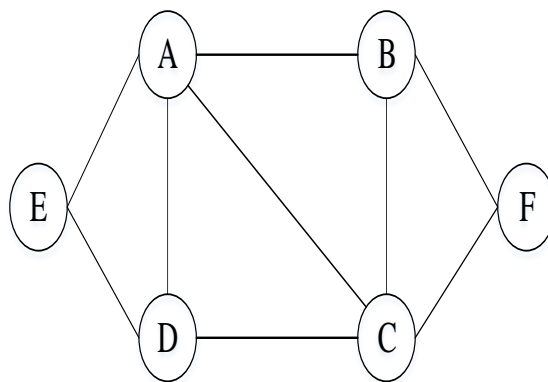


Figure 5: Uncolored graph

The degree of each vertex was found and it is represented below in table.1.

Table 1: Degree of each vertex

Vertex	Valence
A	4
B	3
C	4
D	3
E	2
F	2

After the sorting of vertex according to the descending order of degree value it is represented as *ACBDEF* . Therefore the vertex with highest degree value is colored first and the second highest is colored with second color and so on. The graph after coloring using Welsh-Powell coloring algorithm is shown below in fig.6.

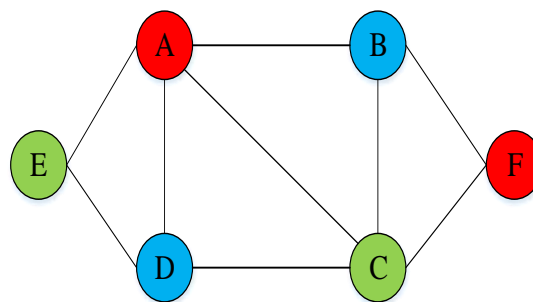


Figure 6: Colored graph

After the coloring process using the process of Welsh-Powell, the graph looks like this. Therefore the chromatic number of the above graph is 3.

2. Conclusion

The objective of this paper was achieved successfully.

5. References

- [1]KauffmanA,Introductiona la Theorie des Sous-ensemblesFlous, VoI.I, MassonetCie, (1973),.
- [2]Zadeh L.A, Similarity Relations and FuzzyOrdering, Inform. Sci. 3 (1971),177-200
- [3] RosenfeldA; FuzzyGraphs, In Fuzzy Sets and their Applications to Cognitive and Decision Processes, Zadeh. L.A., Fu, K.S., Shimura, M., Eds ; Academic Press, New York (1975) 77-95,
- [4] Yeh R. T. Bang S. Y. Fuzzy Relations, FuzzyGraphs and their Applications to Clustering Analysis, In Fuzzy Sets and their Applications to Cognitive and Decision Processes, Zadeh, L.A., Fu,'K.S., Shimura, M., Eds; Academic Press, New York (1975) 125-149.
- [5] George J.Klir, Bo yuan Fuzzy Set and Fuzzy logic Theory and applications, 2003 : 418-465
- [6] Bershtein L, Bozhenuk A. A color problem for fuzzy graph. Lecture notes in computer science. 2001 Jan 1:500-5.