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Analysis of Repair Rate Using Different Modes of Instruction

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Abstract:

In this paper, we have studied about a two-unit cold standby system which is in repair state before and after instruction. It is assumed that every repairman can repair the failed unit with some perfection and in case if unit is not repaired, then another expert repairman is called to repair the unit. There may be some cases in real life when the repairman may not completely repair the failed unit then there may be some instruction from the expert's side to repair the failed unit. In this paper, we have studied the same and using the mathematical concepts we have found MTSF, Availability, Busy period of the repairman, Busy period of the expert repairman (instruction time analysis), Expected number of visits by the expert repairman, Cost benefit analysis.

Keywords: Failed unit, repaired unit, repairman, expert repairman, instruction, MTSF, Availability.

1. Introduction

In the study of reliability, a lot of work has been done on different types of one or two-unit standby system as per their use in the modern industry and business system. The standby unit can be explained in two ways: warm standby unit and cold standby unit. Cold standby means that the redundant units cannot fail while they are waiting, and the warm standby means that the inactive component can fail at the standby state. An example of a cold standby unit is a Stephaney and an example of a warm standby is invertor. Many of the researchers have worked on various concepts like repair time, operating time and rest period, availability, two types of repair facilities, regenerative point technique, failure due to error, repair before and after instruction, etc.

S.K. Gupta (2016) (2017) compared two different cold standby systems, depending on whether or not an assistant repairman needs instructions to fix a malfunctioning equipment. Under the presumption that the standby unit is subpar and is replaced upon failure, two models with different unit cold standby systems with training time, replacement, and preventative maintenance were analysed. R. Gupta (2017) (2018) took into account the instruction time and the possibility of ordinary repairman damaging the unit to the extent it gets more degraded or even irreparable. She analyzed a two-unit cold standby using semi-Markov process and regenerative point technique and derived various measures of system effectiveness. A.K. Taneja (2018) conducted a study on a two-unit cold standby system that had both regular and visiting repairmen. When a unit broke down, the system's regular repairman fixed it; he never left the system. In the case, the regular repairman might grow weary of working on the malfunctioning item, an outside professional repairman would discuss the nature of the defect over the phone and, based on that discussion, either came himself or sent an assistant (an ordinary repairman).

Different system effectiveness metrics were acquired, and profit was assessed in a certain scenario. Renu & P. Bhatia (2019) analyzed two cases of high pressure die casting machine where the secondary units work up to maximum allowable operative time with two repair facility. In every scenario conceivable, the mean time to system failure, steady state, availabilities, anticipated number of system visits, system behaviour, and profit function have been examined.

2. Description of model and Assumptions:

i. System is made up of two identical units. One unit is operating and the other is kept as cold standby.
ii. If there is a failure in one unit, the standby unit will be in operation automatically and the failed unit will go under repair.
iii. In case of failure of both the units, the system will be in failed state.
iv. After the repair, the failed unit will behave like new one.
v. The time to failure for each unit is in exponential distribution and the repair time and instruction time are in arbitrary distribution.
vi. All the random variables are mutually independent.

3. Nomenclature

p Probability that the repairman repairs system without instructions

q Probability that the repairman fails to repair system without instructions

λ Constant failure rate of the operative unit

O operative unit

CS cold standby

$g_1(t)$ p.d.f. of time to repair by repairman before instructions are given

$g_2(t)$ p.d.f. of time to repair by repairman after instructions are given

$G_1(t)$ C.d.f. of time to repair by repairman before instructions are given

$G_2(t)$ c.d.f. of time to repair by repairman after instructions are given

$i(t)$ p.d.f. when expert gives instructions to the repairman

I(t) c.d.f. when expert gives instructions to the repairman

F_{bi} failed unit under the repair of repairman before getting instructions

F_{ai} failed unit under the repair of repairman after getting instructions

F_{BI} repair by the repairman is continuing from the previous state before getting instructions

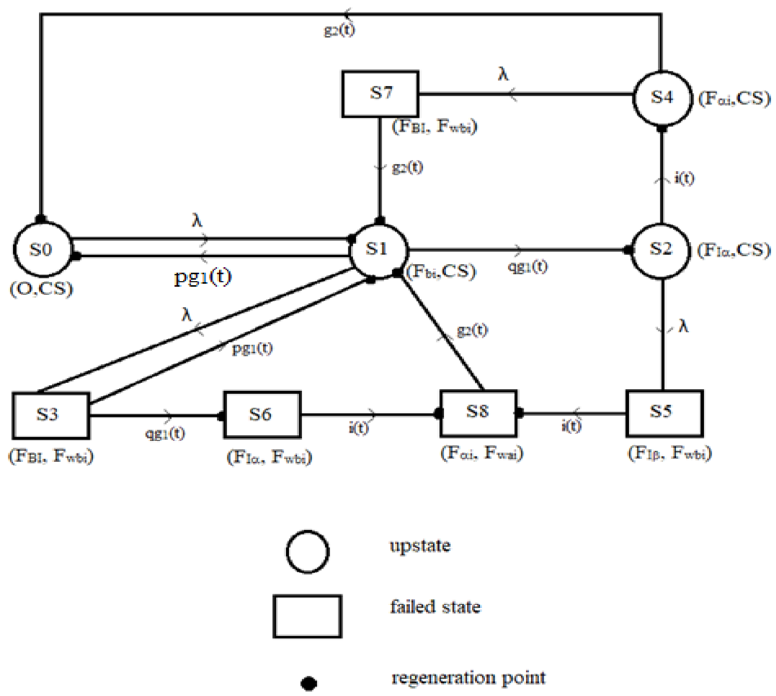
F_{AI} repair by the repairman is continuing from the previous state after getting instructions

$F_{I\alpha}$ expert is giving instructions to the repairman

$F_{I\beta}$ instructions by the expert are continuing from the previous state

F_{wbi} failed unit waiting for repair before getting instructions

F_{wai} failed unit waiting for repair after getting instructions



4. Transition probabilities

The transition probabilities are:

$dQ_{01}(t) = \lambda e^{-\lambda t} dt$

$dQ_{10}(t) = p g_1(t) e^{-\lambda t} dt$

$dQ_{12}(t) = q g_1(t) e^{-\lambda t} dt$

$dQ_{13}(t) = \lambda e^{-\lambda t} \overline{G_1(t)} dt$

$dQ_{16}^{(3)}(t) = \left(\lambda e^{-\lambda t} © q\right) g_1(t) dt = q\left(1 - e^{-\lambda t}\right) g_1(t) dt$

$dQ_{11}^{(3)}(t) = \left(\lambda e^{-\lambda t} © p\right) g_1(t) dt = p\left(1 - e^{-\lambda t}\right) g_1(t) dt$

$dQ_{24}(t) = i(t) e^{-\lambda t} dt$

$dQ_{28}^{(5)}(t) = \left(\lambda e^{-\lambda t} © 1\right) i(t) dt = \left(1 - e^{-\lambda t}\right) i(t) dt$

$dQ_{25}(t) = \lambda e^{-\lambda t} \overline{I(t)} \mathrm{dt}$

$dQ_{40}(t) = g_2(t) e^{-\lambda t} dt$

$dQ_{41}^{(7)}(t) = \left(\lambda e^{-\lambda t} © 1\right) g_2(t) dt = \left(1 - e^{-\lambda t}\right) g_2(t) dt$

$dQ_{47}(t) = \lambda e^{-\lambda t} \overline{G_2(t)} dt$

$dQ_{68}(t) = i(t) dt$

$dQ_{81}(t) = g_2(t)dt$ ....(1-14)

The non-zero elements p_{ij} are as follows:

$p_{01} = 1$, $p_{10} = pg_1^*(\lambda)$, $p_{12} = qg_1^*(\lambda)$, $p_{13} = 1 - g_1^*(\lambda)$, $p_{16}^{(3)} = q(1 - g_1^*(\lambda))$, $p_{11}^{(3)} = p(1 - g_1^*(\lambda))$, $p_{24} = i^*(\lambda)$, $p_{28}^{(5)} = 1 - i^*(\lambda)$, $p_{25} = 1 - i^*(\lambda)$, $p_{40} = g_2^*(\lambda)$, $p_{41}^{(7)} = 1 - g_2^*(\lambda), p_{47} = 1 - g_2^*(\lambda)$, $p_{68} = p_{81} = 1$

From the transition probabilities, it can be verified that

$p_{01} = 1$

$p_{10} + p_{12} + p_{13} = p_{10} + p_{12} + p_{16}^{(3)} + p_{11}^{(3)} = 1$

$p_{24} + p_{25} = p_{24} + p_{28}^{(5)} = 1$

$p_{40} + p_{47} = p_{40} + p_{41}^{(7)} = 1$

$p_{68} = p_{81} = 1$ ……(15-19)

5. Mean Sojourn Time

If T denotes mean sojourn time in state 0, then

$\mu_0 = \int P(T > t)dt = \frac{1}{\lambda}$, $\mu_1 = \frac{1-g_1^*(\lambda)}{\lambda}$, $\mu_2 = \frac{1-i^*(\lambda)}{\lambda}$, $\mu_4 = \frac{1-g_2^*(\lambda)}{\lambda}$,

$\mu_6 = \int_0^\infty \overline{I(t)}dt = \int_0^\infty t dI(t) = \text{Mean instruction time}$, $\mu_8 = \int_0^\infty \overline{G_2(t)}dt$ ………(20)

The unconditional mean time taken by the system to transit to any regenerative state i when it is counted from the epoch of entrance into that state is, mathematically, stated as

$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -\frac{d}{ds} q_{ij}^*|_{s=0}$ ………(21)

Thus, $m_{01} = \mu_0$, $m_{10} + m_{12} + m_{13} = \mu_1$, $m_{24} + m_{25} = m_{24} + m_{28}^{(5)} = \mu_2$, $m_{40} + m_{47} = m_{40} + m_{41}^{(7)} = \mu_4, m_{10} + m_{12} + m_{11}^{(3)} + m_{16}^{(3)} = k_1$ ………(22)

6. Mean time to system failure

To determine the MTSF of the system, we regard the failed states of the system as absorbing. By probabilistic arguments, we have

$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t)$

$\phi_1(t) = Q_{13}(t) + Q_{12}(t) Ⓢ \phi_2(t) + Q_{10}(t) Ⓢ \phi_0(t)$

$\phi_2(t) = Q_{25}(t) + Q_{24}(t) Ⓢ \phi_4(t)$

$\phi_4(t) = Q_{47}(t) + Q_{40}(t) Ⓢ \phi_0(t)$ ……(23-26)

Taking the L.S.T. of the equation (23-26) and solving them for $\phi_0^{**}(s)$, we have

$$\phi_0^{**}(s) = \frac{Q_{01}^{**}(s)[Q_{13}^{**}(s)+Q_{12}^{**}(s)Q_{25}^{**}(s)+Q_{12}^{**}(s)Q_{24}^{**}(s)Q_{47}^{**}(s)]}{1-Q_{01}^{**}(s)[Q_{10}^{**}(s)+Q_{12}^{**}(s)Q_{24}^{**}(s)Q_{40}^{**}(s)]} \quad \ldots\ldots\ldots(27)$$

Now the MTSF, given that the system started at the beginning of state 0 is

$$T_0 = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D} \quad \ldots\ldots\ldots(28)$$

Where $N = \mu_{0+}\mu_1 + P_{12}\mu_2 - P_{12}p_{24} + P_{12}p_{24}\mu_4$ ………(29)

and $D = p_{01} - p_{10} - p_{12}p_{24}p_{40}$ ………(30)

7. Availability Analysis

$M_i(t)$ denotes the probability that the system starting in up regenerative state is up at time t without passing through any regenerative state.

Thus, we have $M_0(t) = e^{-\lambda t}$, $M_1(t) = e^{-\lambda t}\overline{G_1(t)}$, $M_2(t) = e^{-\lambda t}\overline{I(t)}$, $M_4(t) = e^{-\lambda t}\overline{G_2(t)}$ ………(31)

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{12}(t)©A_2(t) + q_{11}^{(3)}(t)©A_1(t) + q_{16}^{(3)}(t)©A_6(t)$$

$$A_2(t) = M_2(t) + q_{24}(t)©A_4(t) + q_{28}^{(5)}(t)©A_8(t)$$

$$A_4(t) = M_4(t) + q_{40}(t)©A_0(t) + q_{41}^{(7)}(t)©A_1(t)$$

$$A_6(t) = q_{68}(t)©A_8(t)$$

$$A_8(t) = q_{81}(t)©A_1(t) \quad \ldots\ldots(32\text{-}37)$$

Taking Laplace transform of equation (31) and solving for $s \to 0$, we get

$M_0^*(0) = \mu_0$, $M_1^*(0) = \mu_1$, $M_2^*(0) = \mu_2$, $M_4^*(0) = \mu_4$ ………(38)

The steady state availability of the system is given by

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1} \quad \ldots\ldots\ldots(39)$$

Where $N_1 = \mu_0\left[1 - p_{12}p_{28}^{(5)} - p_{16}^{(3)} - p_{11}^{(3)} - p_{12}p_{24}p_{41}^{(7)}\right] + \mu_1 + \mu_2 p_{12} + \mu_4 p_{12}p_{24}$ ……(40)

and $D_1 = \mu_0[p_{10} + p_{12}p_{24}p_{40}] + k_1 + \mu_2 p_{12} + \mu_4 p_{12}p_{24} + \mu_8\left[p_{12}p_{28}^{(5)} + 2p_{16}^{(3)}\right]$ ……(41)

8. Busy period analysis

Busy period of the repairman:

$$B_0^r(t) = q_{01}(t)©B_1^r(t)$$

$$B_1^r(t) = W_1(t) + q_{10}(t)©B_0^r(t) + q_{12}(t)©B_2^r(t) + q_{11}^{(3)}(t)©B_1^r(t) + q_{16}^{(3)}(t)©B_6^r(t)$$

$$B_2^r(t) = q_{24}(t)©B_4^r(t) + q_{28}^{(5)}(t)©B_8^r(t)$$

$B_4^r(t) = W_4(t) + q_{40}(t)©B_0^r(t) + q_{41}^{(7)}(t)©B_1^r(t)$

$B_6^r(t) = q_{68}(t)©B_8^r(t)$

$B_8^r(t) = W_8(t) + q_{81}(t)©B_1^r(t)$ ……(42-47)

Where $W_1(t) = e^{-\lambda t}\overline{G_1(t)} + [\lambda e^{-\lambda t}©1]\,\overline{G_1(t)}$, $W_4(t) = e^{-\lambda t}\overline{G_2(t)} + [\lambda e^{-\lambda t}©1]\,\overline{G_2(t)}$, $W_8(t) = \overline{G_2(t)}$ ………(48)

Taking Laplace transform of equation (48) and solving for $s \to 0$, we get

$W_1^*(0) = k_1$, $W_4^*(0) = \mu_4 = W_8^*(0)$ ………(49)

In the steady-state, the total fraction of time for which repairman is busy is

$B_0^r = \lim_{s\to 0} s\, B_0^{r^*} = \frac{N_2}{D_1}$ ………(50)

Where $N_2 = \mu_4\left[p_{12}p_{28}^{(5)} + p_{16}^{(3)} + p_{12}p_{24}\right] + k_1$ ………(51)

And D_1 is same as in equation (41).

Busy period analysis for expert repairman/ Instruction time analysis

$B_0^i(t) = q_{01}(t)©B_1^i(t)$

$B_1^i(t) = q_{10}(t)©B_0^i(t) + q_{12}(t)©B_2^i(t) + q_{11}^{(3)}(t)©B_1^i(t) + q_{16}^{(3)}(t)©B_6^i(t)$

$B_2^i(t) = W_2(t) + q_{24}(t)©B_4^i(t) + q_{28}^{(5)}(t)©B_8^i(t)$

$B_4^i(t) = q_{40}(t)©B_0^i(t) + q_{41}^{(7)}(t)©B_1^i(t)$

$B_6^i(t) = W_6(t) + q_{68}(t)©B_8^{ii}(t)$

$B_8^i(t) = q_{81}(t)©B_1^i(t)$ ……(52-57)

Where $W_2(t) = e^{-\lambda t}\overline{I(t)} + [\lambda e^{-\lambda t}©1]\,\overline{I(t)}$, $W_6(t) = \overline{I(t)}$ ………(58)

Taking Laplace transform of equation (58) and solving for $s \to 0$, we get

$W_2^*(0) = \mu_2 = W_6^*(0)$ ………(59)

In steady state, the total fraction of time for which the expert repairman is busy in giving instructions is

$B_0^i = \lim_{s\to 0} s\, B_0^{i^*} = \frac{N_3}{D_1}$ ………(60)

Where$N_3 = \mu_2\left[p_{12} + p_{16}^{(3)}\right]$ ………(61)

And D_1 is same as in equation (41).

9. Expected number of visits by expert repairman

$V_0(t) = Q_{01}(t)Ⓢ V_1(t)$

$V_1(t) = Q_{10}(t)Ⓢ V_0(t) + Q_{12}(t)Ⓢ[1 + V_2(t)] + Q_{16}^{(3)}(t)Ⓢ[1 + V_6(t)] + Q_{11}^{(3)}(t)Ⓢ V_1(t)$

$V_2(t) = Q_{24}(t)Ⓢ V_4(t) + Q_{28}^{(5)}(t)Ⓢ V_8(t)$

$V_4(t) = Q_{40}(t)Ⓢ V_0(t) + Q_{41}^{(7)}(t)Ⓢ V_1(t)$

$V_6(t) = Q_{68}(t)Ⓢ V_8(t)$

$V_8(t) = Q_{81}(t)Ⓢ V_1(t)$ ……(62-67)

In steady-state, the number of visits per unit time is given by taking $s \to 0$ and $t \to \infty$

$V_0 = \lim_{t\to\infty} \frac{V_0(t)}{t} = \lim_{s\to\infty}[sV_0^{**}(s)] = \frac{N_4}{D_1}$ ………(68)

Where $N_4 = p_{12} + p_{16}^{(3)}$ ………(69)

And D_1 is same as in equation (41).

10. Cost-Benefit analysis

The expected cost-profit of the system in steady state is given by

$P_1 = C_0A_0 - C_1B_0^r - C_2B_0^i - C_3V_0$ ………(70)

Where C_0 is revenue per unit up-time of the system.

C_1 is cost per unit time for which the assistant repairman is busy.

C_2 is cost per unit for which expert repairman is busy.

C_3 is cost per visit of expert repairman.

11. Comparison Analysis

MTSF vs. Failure Rate of the Main Unit with Initial State S0

The MTSF has been determined by taking different values of the failure rate (λ) of the operative unit as shown in Table 1.1 and the graphs corresponding to these cases have been shown in Figure 1. This has been done by taking specific values of the probability of the system that needs repair (p_{12}) & the probability that it needs replacement (p_{24}) for the different cases (i) $p_{12} > p_{24}$ ($p_{12} = 0.75$; $p_{24} = 0.25$) (ii) $p_{12} = p_{24}$ ($p_{12} = 0.50$; $p_{24} = 0.50$) (iii) $p_{12} < p_{24}$ ($p_{12} = 0.25$; $p_{24} = 0.75$).

For any fixed value of p_{12}/p_{24}, Table 1.1 and Figure 1 shows that the system's MTSF (T_0) rapidly reduces when the operating unit's failure rate (λ) increases. The percentage decrease in MTSF has been reported to range from 75% to 18% approximately when λ changes between 0.0015 and 0.0095. Notably, this % decrease in MTSF is nearly identical in all three examples. In contrast, at a given value of the operating unit's failure rate (λ), MTSF falls with the likelihood of system repair decreasing or the likelihood of replacement increasing. The value of MTSF reduces by 26.34% for λ = 0.0095 and by 28.22% for λ = 0.001 when p12 drops from 0.75 to 0.25.

TABLE- 1.1

MTSF T_0 vs. Failure Rate λ				
Sr. No.	λ	**T_0** **$p_{12} = 0.75$; $p_{24} = 0.25$**	**T_0** **$p_{12} = 0.50$; $p_{24} = 0.50$**	**T_0** **$p_{12} = 0.25$; $p_{24} = 0.75$**
1	0.0015	210618.5000	175999.0312	151194.4062
2	0.0025	53186.5742	44519.4921	38299.5468
3	0.0035	23877.4199	20019.9804	17249.0625
4	0.0045	13566.8603	11394.0937	9831.2968
5	0.0055	8760.6767	7378.1918	6375.3793
6	0.0065	6151.4350	5184.1206	4485.8974
7	0.0075	4565.9667	3853.5878	3339.2756
8	0.0085	3531.2580	2985.1127	2590.3239
9	0.0095	2818.4272	2386.3300	2073.5976

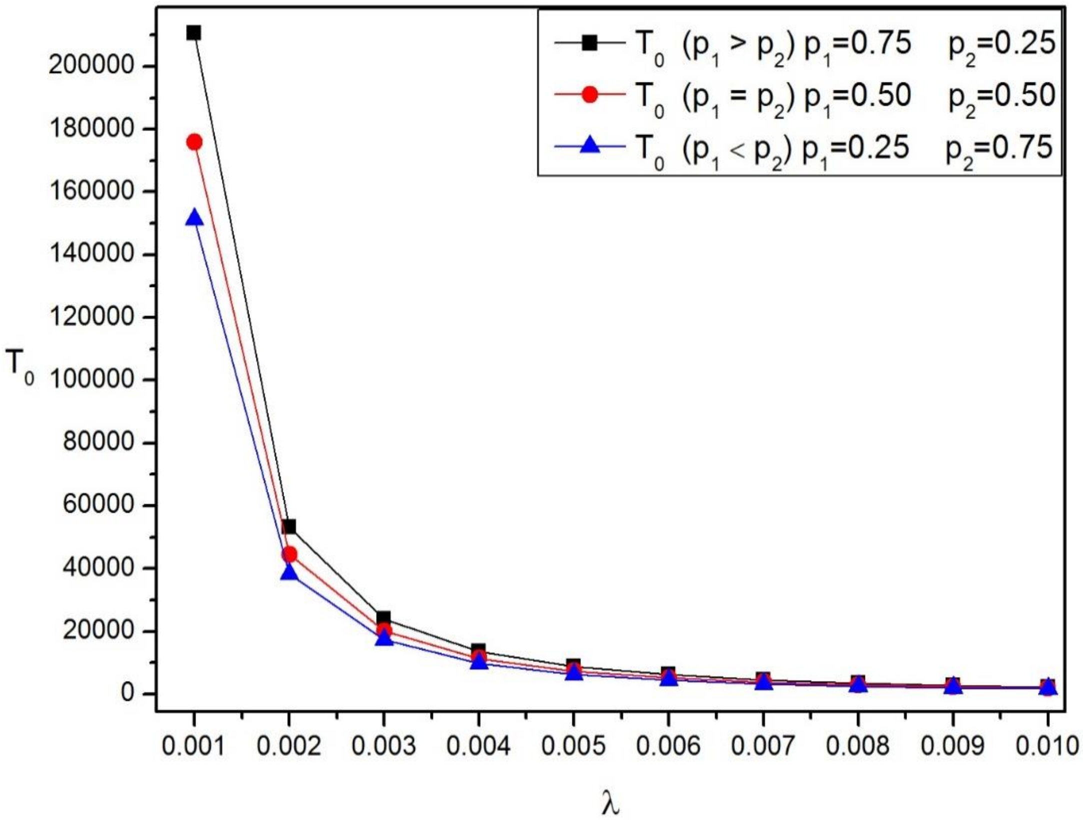


Figure 1 (MTSF T_0 vs. Failure Rate of the system)

MTSF vs. Failure Rate of the Main Unit with Initial State S1

Examining the various scenarios of probability of repair (p_{12}) and replacement (p_{24}) that the system requires, the MTSF has been assessed for the various operative unit failure rate values (λ) as indicated in Table 1.2. The corresponding graphs for these scenarios are displayed in Figure 2.

Table 1.2 and Figure 2 demonstrate that, for any fixed value of p_{12}/p_{24}, the system's MTSF (T_1) rapidly drops when the operating unit's failure rate (λ) increases. It has been noticed that the percentage decrease in MTSF ranges from around 74.85% to 18.51% when λ fluctuates between 0.0015 and 0.0095, and in all three situations, this percentage decrease in T_1 is nearly identical. Conversely, for a given value of λ, T_1 reduces as p_{12} falls or p_{24} increases. The value of T_1 reduces by 27.54% for $\lambda = 0.0095$ and by 28.35% for $\lambda = 0.0015$ when p_{12} drops from 0.75 to 0.25. This indicates that the fluctuations in MTSF (T_1) are less for a given variation in p_{12} at greater failure rates (λ).

TABLE- 1.2

MTSF T_1 vs. Failure Rate λ				
Sr. No.	**λ**	**T_1 $p_{12} = 0.75; p_{24} = 0.25$**	**T_1 $p_{12} = 0.50; p_{24} = 0.50$**	**T_1 $p_{12} = 0.25; p_{24} = 0.75$**
1	0.0015	209909.1562	175267.5625	150438.1718
2	0.0025	52887.3281	44193.6640	37973.8281
3	0.0035	23711.5332	19843.0390	16964.1777
4	0.0045	13467.6542	11286.5927	9617.8310
5	0.0055	8611.4794	7212.3569	6204.7636
6	0.0065	6019.9096	5046.0620	4343.8491
7	0.0075	4452.4931	3735.3701	3317.6325
8	0.0085	3432.0729	2881.7756	2483.9848
9	0.0095	2730.3559	2294.5664	1979.1617

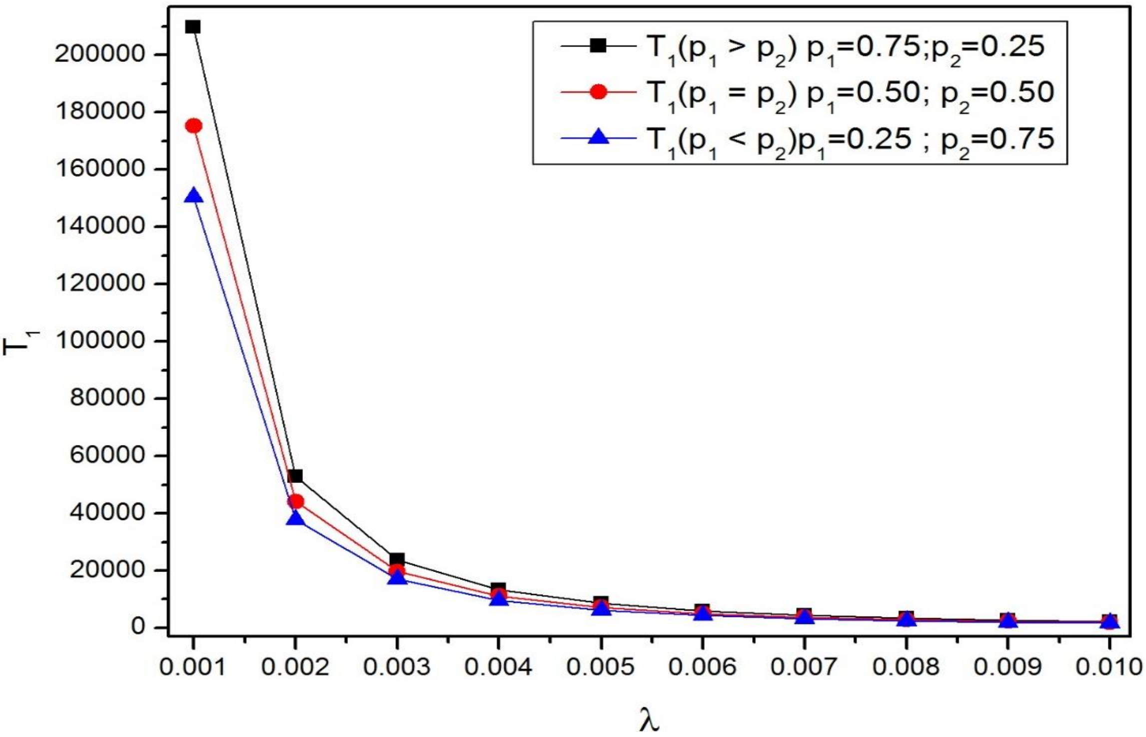


Figure 2 (MTSF T_1 vs. Failure Rate λ of the Main Unit)

12. Conclusion

The MTSF and availability of the two-unit cold standby system, have been obtained easily and quickly by using the path analysis. It has been verified that MTSF being a positional measure, it depends upon the initial state. Whereas the steady state availability of the system being the global measure, has been found to be the same, although determined separately, by assuming the state S0 and the base state S1 as the initial states.

It has further been observed from the analysis of the system that in general all the MTSF of the system w.r.t. S0, S1 (as the initial states), decrease rapidly with the increase in failure rate (λ) of operating unit, for any fixed value of the probability of minor or major failures and the inspection, replacement and repair rates.

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