SINGULARITY ANALYSIS OF INDIGENOUSLY DEVELOPED 6-DOF SPS PARALLEL MANIPULATOR IN ITS CALCULATED WORKSPACE

Gupta, R.K. [1], Mishra, V. K. [2], T., Aravind[3], Sinha, A. K.[4]

[1,2,3,4] Centre for Design and Manufacturing, Bhabha Atomic Research Centre, Mumbai, (India)

ABSTRACT

Striving for greater accuracy and greater performance, some machine manufactures are developing parallel structure for the next generation machine. This necessarties that these machines need to be analyzed for the effects of existing singularities inside or boundary of evaluated workspace of indigenously developed 6-DoF SPS (S: Spherical, P: Prismatic) parallel manipulator here in Bhabha atomic research Centre, Mumbai. Singular configurations are particular poses of the moving platform of manipulator, in which the parallel manipulator becomes uncontrollable and must therefore be prevented during operation. This paper addresses nominal kinematic model and existing singularities in its calculated workspace of selected configuration of 6-DoF fully parallel manipulators. Starting from the inverse nominal kinematic model of parallel manipulator, a Matlab code was written to solve the inverse kinematic model for workspace evaluation of 6-DoF parallel manipulator. Then after inverse Jacobian matrix, which maps the moving platform's position and orientation to the actuated joint velocities are derived. The singularity analysis is transformed into a search for specific geometric conditions inside or boundary of the evaluated workspace, which apply, when the manipulator is in a singular pose. In this paper the checking of singularities has been done within and boundary of usable workspace with the help of determinant of Jacobian matrix. Usable workspace is the subset of total evaluated workspace which is defined by the application of the manipulator system. Furthermore it has been found out that there is no any singularities existing within or in boundary of usable workspace of this selected configuration of 6-DoF SPS parallel manipulator system.

Keywords: Parallel Manipulator, Workspace, Singularities, Jacobian Matrix

I. INTRODUCTION

A Parallel manipulator is very useful and highly accurate six-axis robotic platform. It has attracted attention of many researchers and mathematicians due to its wide range of applications in nuclear, medical, cryogenics and other emerging fields where minimal human involvement, greater automation and better accuracy is required. These mechanisms have closed loop kinematics. Many challenges however, exist in this field as of now from the point of view of configuration, design, and control to the improvement of its accuracy. Lot of research is going on in the aspect of design, configuration and its control. This paper addresses the issue of existing singularities in its calculated workspace of indigenously developed 6- axis parallel manipulator. Workspace is defined as the space reachable by the moving platform of 6-DoF parallel manipulator machine. Workspace shape and its volume are determined by the kind of mechanical architecture and the angular and linear range of displacement of each actuator. Workspace of the parallel manipulator also depends on maximum and minimum length of the actuators. Workspace determination is an important step in the design and kinematic modeling of any parallel manipulator architecture. It plays an important role in the time of singular points determination of the parallel manipulator. **[1]**. In this paper singularity analytical equation has been derived for already calculated workspace by Gupta, R.K., **[2]** for same indigenously developed 6-DoF S$\underline{\text{P}}$S (S: Spherical, P: Prismatic) parallel manipulator.

Singularity problem is one of the fundamental issues of parallel mechanisms. If the moving platform has been placed on a singular position, it would get or lose some extra degrees of freedom in a certain direction **[3, 4]**. In the former case, the pose of the moving platform is uncontrollable; the joint driving force that balances external load on the moving platform will tend to be infinity. As a result, normal operations of the parallel mechanisms are affected seriously; even the structure of the mechanism is destroyed. In the latter case, the motion of the moving platform along the certain directions cannot be accomplished.

Either in the designing of parallel mechanism or in motion controlling, the singularity is an important problem that could not be avoided. Although the singularity problems of parallel mechanisms were studied early in 1990s, this problem is very difficult; there are not general conclusions about it so far. In the mid-2000s, some scholars **[5, 6]** found that, in the workspace the singular configurations were not the isolated points. They even found a series of singular configurations called the singular locus, which changed consecutively. There are some approaches, which can be used to do singularity research, such as screw theory, Grassman geometry, and matrix analysis. Matrix analysis method is a widely applied method. Whether the singularity of parallel mechanism occurs in some certain postures or not could be determined bycalculating the determinant of the Jacobian matrix. Singular configurations are the roots of the nonlinear equation, which corresponds to the determinant of the Jacobian matrix.

Our strategy is based on matrix analysis method to avoid the singularities is to analyze the workspace with regard to the singular poses in advance and subsequently steering clear of them. The analysis of singular poses is partly based on the checking for regularity of the inverse Jacobian matrix J^{-1}, which relates the actuated joint velocities to the translation and rotation of the moving platform and is usually pose dependent. Weather the singularity of parallel manipulator occurs in some certain postures or not could be determined by calculating the determinant of J. When the determinant J is equal to zero, singularities configuration will occur. Ma and Angeles **[7]** classified the kinematic singularities of a closed-loop mechanism into three categories: architecture, configuration and formulation singularity. While architecture singularity is related to a particular architecture of a parallel manipulator, formulation singularity is due to a bad modelling method. The configuration singularities can be identified in that the instantaneous relationship between the vectors of the moving platform velocity ($\dot{\Pi}$) and the actuated joint velocity ($\dot{l}$) is established as follows: A ($\dot{\Pi}$) = B ($\dot{l}$) Configuration

singularity is further classified into three different categories: forward, inverse and combined singularity. The first, second and third kind of configuration singularities occur respectively when matrix A is singular, matrix B is singular and both matrices are simultaneously singular **[8].**

II NOMINAL MODEL OF PARALLEL MANIPULATOR

A typical 6-axis parallel manipulator consists of a moving platform that is connected to a stationary base through six numbers of parallel linear independent actuators with the help of spherical and /or universal joints at the ends **[9]** as shown in Fig. 1 and Fig. 2. This machine has been developed in centre for design and manufacturing, BARC (Mumbai), India.

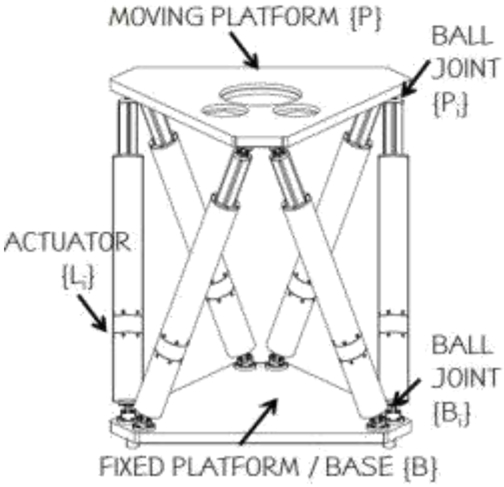

Fig. 1 Six-Axis Parallel Manipulator

This machine is capable to move in three linear directions and as well as in three angular directions in 3-D space. A very promising application of this six-axis machine is in the positioning and orientation of instrumentation in synchrotron radiation beam lines in RRCAT (Indore), India. Some of other applications are Tool Control for Precision-machining & Manufacturing, Positioning of Optics.

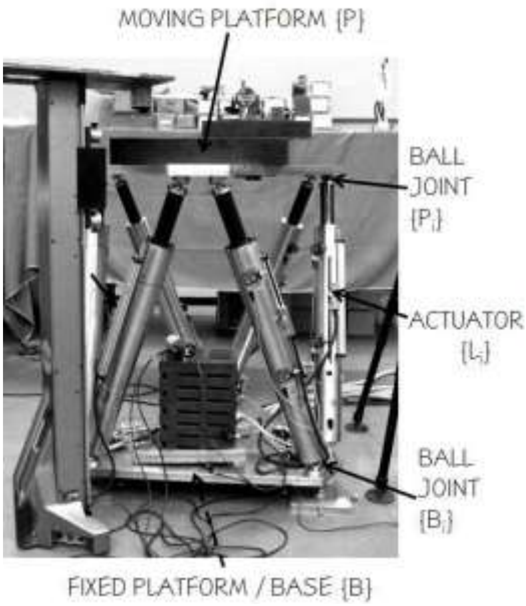

Fig. 2 Six-axis Parallel Manipulator Developed at CDM, BARC

The nominal kinematic model of the parallel manipulator is an ideal model which is based on following assumptions; 1) All the ball joints can be treated as points and the links as straight lines of known length which are connected between the joint centers. 2) The actuators are perfectly assembled to the joints so that each actuator axis passes through the respective joint centers. 3) The platform structure is rigid and the locations of the joints are precisely known. 4) There is no deformation in the actuators and other components **[10] [11].**

The joint pairs attached to the moving plate and the base plates are denoted by P_1 to P_6 and B_1 to B_6 respectively. It is assumed that the spherical joints on the base and moving plate form a circle as shown in Fig. 3. Refer to

Fig. 3 and Fig. 4, coordinate frames {B} and {P} are arbitrary embedded in the centre of base and centre of the platform with vectors referenced in coordinate frame {B} denoted by B_v and vectors referenced in {P} denoted by P_v.Z-axis is perpendicular to the base plane. The joints on the base are denoted by B_i and the position of the joint centers with respect to (B} are given by the vectors ^Bb_i. Similarly P_i denotes movable platform joints with the location of the joint centers given by ^Pp_i (or ^Bu_i in the base frame). The six legs are denoted by L_i with a vector from B_i to P_i defined by $\lambda_i{}^Bl_i$. Where λ_i is the magnitude (the length of the leg) and ^Bl_i is a directional unit vector. Each leg contains a length monitoring device as a linear scale so λ_i that is a measurable quantity. A vector Λ containing all λ_i will be used to describe the measured leg lengths through a displacement vector ^Bq and through a rotation vector ^BR_p **[9]**.

The vector $^Bq= [P_x\ P_y\ P_z]^T$ gives the relative displacement of the origin of {P} from the origin of {B}. Where P_x, P_y, P_z are the translation motion of moving platform in x, y and z direction respectively. Rotation matrix ^BR_P is formed using the roll-pitch-yaw angles α, β and γ respectively.

So finally an orientation vector can be defined by $\Omega= [\alpha\ \beta\ \gamma]^T$ where C, S stands for cosine and sine respectively. Rotation matrix ^BR_P is obtained from the three Euler angles α, β and γ **[9]**. The inverse kinematics deals with calculating the leg lengths when the position and orientation of moving platform is given.

$\Omega = {}^BR_P = Rot\ (z, \gamma).Rot\ (y, \beta).Rot(x, \alpha) =$

$$\begin{bmatrix} C\gamma\ C\beta & C\gamma\ S\beta\ S\alpha - S\gamma\ C\alpha & C\gamma\ S\beta\ C\alpha + S\gamma S\alpha \\ S\gamma\ C\beta & S\gamma\ S\beta\ S\alpha + C\gamma\ C\alpha & S\gamma\ S\beta\ C\alpha - C\gamma S\alpha \\ -S\beta & C\beta\ S\alpha & C\beta\ C\alpha \end{bmatrix}$$

In effect, it maps global pose to local actuator lengths i.e. Π to Λ. Where $\Pi = [P_x, P_y, P_z, \alpha, \beta \text{ and } \gamma]$ is the pose of the moving platform. With the assumptions of the nominal model, the inverse kinematics are uncomplicated and yielding a closed form solution. After noticing that the platform joint location vectors are related by $^Bu_i = {}^BR_p{}^Pp_i$, the vector chain in Fig. 4 can be written as

$$\lambda_i{}^Bl_i = {}^Bu_i + {}^Bq - {}^Bb_i \qquad (1)$$

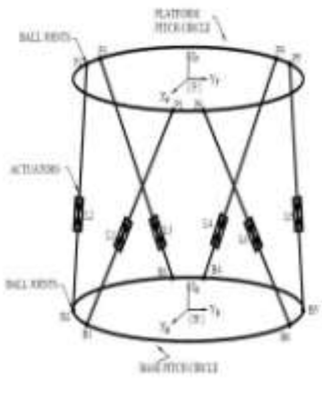

Fig.3 Nominal Model of the 6-axis Parallel Manipulator

The nominal inverse kinematics are then solved by taking the magnitude of equation (1)

$$\lambda_i = |\lambda_i{}^{B}l_i| = |{}^{B}u_i + {}^{B}q - {}^{B}b_i| \qquad (2)$$

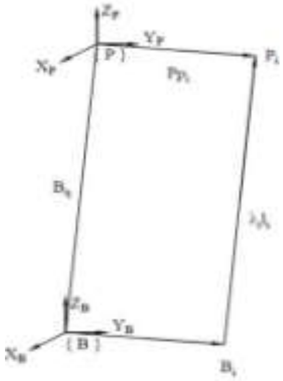

Fig.4 Nominal Vector Chain for i^{th} Leg

III. SINGULARITY ANALYSIS OF SELECTED 6-DOF PARALLEL MANIPULATOR CONFIGURATION

A MATLAB code was written to solve the inverse kinematic model as shown in equation 2 for workspace evaluation of this selected 6-SPS parallel manipulator. MATLAB which consists of dimensional parameters of both the platform and the minimum and maximum length of the linear actuators. After the evaluation of total workspace by solving the inverse kinematic model with the help of MATLAB code, Jacobian matrix has been formulated to find out the singular position in usable workspace rather than the total workspace. Usable workspace is the subset of the total workspace. Usable workspace depends on application of the system. In this paper the translational usable workspace is ± 100 mm along all three axis's while the rotational usable workspace is ± 10 degree about all three directions has been taken.

3.1 Jacobian Formulation: 6 SPS Parallelmanipulator

Inverse kinematic analysis primarily takes the orientation angles and translation value (P_x, P_y, P_z, α, β and γ) as inputs and gives the required legs length of actuators to achieve that position and orientation. It does not give any idea about, what would be the possible range of the angles and translation value that can be attained by the manipulator. This depends on the various parameters of the configuration and is also constrained by actuator and joints limits. Jacobian analysis helps in finding singularities and tells us, if the given orientation can be achieved by the given manipulator configuration or not. It tells us about all possible poses of manipulator. The Jacobian is a multidimensional form of the derivative. In robotics Jacobians are mainly used to relate output velocities to joint velocities. The inverse kinematics is the mapping from Π to Λ. For a given position and orientation of the center of the upper platform (i.e. P_x, P_y, P_z, α, β and γ) in Cartesian space, the lengths of the legs L_i are determined by

$$L_1^2 = ((X_{P1})_b - d/2\sqrt{3} - b/\sqrt{3})^2 + ((Y_{P1})_b - d/2)^2 + ((Z_{P1})_b)^2 \qquad (3)$$

$$L_2^2 = ((X_{P2})_b - d/2 + b/2)^2 + ((Y_{P2})_b - d/2 - b/2)^2 + ((Z_{P2})_b)^2 \qquad (4)$$

$$L_3^2 = ((X_{P3})_b + d/\sqrt{3} + b/2\sqrt{3})^2 + ((Y_{P3})_b - b/2)^2 + ((Z_{P3})_b)^2 \quad (5)$$

$$L_4^2 = ((X_{P4})_b + d/\sqrt{3} + b/2\sqrt{3})^2 + ((Y_{P4})_b + b/2)^2 + ((Z_{P4})_b)^2 \quad (6)$$

$$L_5^2 = ((X_{P5})_b - d/2\sqrt{3} + b/2\sqrt{3})^2 + ((Y_{P5})_b + d/2 + b/2)^2 + ((Z_{P5})_b)^2 \quad (7)$$

$$L_6^2 = ((X_{P6})_b - d/2\sqrt{3} - b/\sqrt{3})^2 + ((Y_{P6})_b + d/2)^2 + ((Z_{P6})_b)^2 \quad (8)$$

Where

$$(X_{P1})_b = C\gamma\, C\beta\, (X_{P1})_P + (C\gamma\, S\beta\, S\alpha - S\gamma\, C\alpha)(Y_{P1})_P + (C\gamma\, S\beta\, C\alpha + S\gamma\, S\alpha)(Z_{P1})_P + P_x \quad (9)$$

$$(X_{P2})_b = C\gamma\, C\beta\, (X_{P2})_P + (C\gamma\, S\beta\, S\alpha - S\gamma\, C\alpha)(Y_{P2})_P + (C\gamma\, S\beta\, C\alpha + S\gamma\, S\alpha)(Z_{P2})_P + P_x \quad (10)$$

$$(X_{P3})_b = C\gamma\, C\beta\, (X_{P3})_P + (C\gamma\, S\beta\, S\alpha - S\gamma\, C\alpha)(Y_{P3})_P + (C\gamma\, S\beta\, C\alpha + S\gamma\, S\alpha)(Z_{P3})_P + P_x \quad (11)$$

$$(X_{P4})_b = C\gamma\, C\beta\, (X_{P4})_P + (C\gamma\, S\beta\, S\alpha - S\gamma\, C\alpha)(Y_{P4})_P + (C\gamma\, S\beta\, C\alpha + S\gamma\, S\alpha)(Z_{P4})_P + P_x \quad (12)$$

$$(X_{P5})_b = C\gamma\, C\beta\, (X_{P5})_P + (C\gamma\, S\beta\, S\alpha - S\gamma\, C\alpha)(Y_{P5})_P + (C\gamma\, S\beta\, C\alpha + S\gamma\, S\alpha)(Z_{P5})_P + Px \quad (13)$$

$$(X_{P6})_b = C\gamma\, C\beta\, (X_{P6})_P + (C\gamma\, S\beta\, S\alpha - S\gamma\, C\alpha)(Y_{P6})_P + (C\gamma\, S\beta\, C\alpha + S\gamma\, S\alpha)(Z_{P6})_P + P_x \quad (14)$$

$$(Y_{P1})_b = S\gamma\, C\beta\, (X_{P1})_P + (S\gamma\, S\beta\, S\alpha + C\gamma\, C\alpha)$$

(20)

$$(Z_{P1})_b = C\beta\, S\alpha\, (Y_{P1})_P + C\beta\, C\alpha\, (Z_{P1})_P - S\beta\, (X_{P1})_P + P_z \quad (21)$$

$$(Z_{P2})_b = C\beta\, S\alpha\, (Y_{P2})_P + C\beta\, C\alpha\, (Z_{P2})_P - S\beta\, (X_{P2})_P + P_z \quad (22)$$

$$(Z_{P3})_b = C\beta\, S\alpha\, (Y_{P3})_P + C\beta\, C\alpha\, (Z_{P3})_P - S\beta\, (X_{P3})_P + P_z \quad (23)$$

$$(Z_{P4})_b = C\beta\, S\alpha\, (Y_{P4})_P + C\beta\, C\alpha\, (Z_{P4})_P - S\beta\, (X_{P4})_P + P_z \quad (24)$$

$$(Z_{P5})_b = C\beta\, S\alpha\, (Y_{P5})_P + C\beta\, C\alpha\, (Z_{P5})_P - S\beta\, (X_{P5})_P + P_z \quad (25)$$

$$(Z_{P6})_b = C\beta\, S\alpha\, (Y_{P6})_P + C\beta\, C\alpha\, (Z_{P6})_P - S\beta\, (X_{P6})_P + P_z \quad (26)$$

From the summary of the previous section it can be seen that equation (3) - (8) give the explicit expression of the lengths of the legs L_i in terms of the coordinates of the vertices of the upper platform (X_{Pi} , Y_{Pi}, Z_{Pi}) and then

$(Y_{P1})_P + (S\gamma\, S\beta\, C\alpha - C\gamma\, S\alpha)\,(Z_{P1})_P + P_y$ (15)

$(Y_{P2})_b = S\gamma\, C\beta\,(X_{P2})_P + (S\gamma\, S\beta\, S\alpha + C\gamma\, C\alpha)$
$(Y_{P2})_P + (S\gamma\, S\beta\, C\alpha - C\gamma\, S\alpha)\,(Z_{P2})_P + P_y$ (16)

$(Y_{P3})_b = S\gamma\, C\beta\,(X_{P3})_P + (S\gamma\, S\beta\, S\alpha + C\gamma\, C\alpha)$
$(Y_{P3})_P + (S\gamma\, S\beta\, C\alpha - C\gamma\, S\alpha)\,(Z_{P3})_P + P_y$ (17)

$(Y_{P4})_b = S\gamma\, C\beta\,(X_{P4})_P + (S\gamma\, S\beta\, S\alpha + C\gamma\, C\alpha)$
$(Y_{P4})_P + (S\gamma\, S\beta\, C\alpha - C\gamma\, S\alpha)\,(Z_{P4})_P + P_y$

$\dot{L}$ $\dot{X}$ (18)

$(Y_{P5})_b = S\gamma\, C\beta\,(X_{P5})_P + (S\gamma\, S\beta\, S\alpha + C\gamma\, C\alpha)$
$(Y_{P5})_P + (S\gamma\, S\beta\, C\alpha - C\gamma\, S\alpha)\,(Z_{P5})_P + P_y$ (19)

$(Y_{P6})_b = S\gamma\, C\beta\,(X_{P6})_P + (S\gamma\, S\beta\, S\alpha + C\gamma\, C\alpha)$
$(Y_{P6})_P + (S\gamma\, S\beta\, C\alpha - C\gamma\, S\alpha)\,(Z_{P6})_P + P_y$

Equations (9) - (26) express these coordinates with the given position and orientation of the center of the upper platform (P_x, P_y, P_z, α, β and γ).

Therefore, to obtain the relationship between the link velocity $\dot{L}$ and the velocity in Cartesian space $\dot{\Pi}$ differentiating both sides of Equations (3) – (26) with respect to time yields

$$= J_1(\Pi)\quad {}_P \tag{27}$$

$$_P = J_2(\Pi) \tag{28}$$

where $X_P = [X_{P1}\ Y_{P1}\ Z_{P1}\ \ X_{P2}\ Y_{P2}\ Z_{P2}\ X_{P3}\ Y_{P3}\ Z_{P3}\ X_{P4}\ Y_{P4}\ Z_{P4}\ X_{p5}\ Y_{P5}\ Z_{P5}\ X_{p6}\ Y_{P6}\ Z_{P6}]^T$.

$J_1(\Pi)$ is the (6 x 18) matrix and $J_2(\Pi)$ is the (18 x 6) matrix. It can be seen that J_1 and J_2 are explicit functions of X_P, but implicit functions of
Π. To emphasize that the given Π can uniquely determine the value of J_1 and J_2, an explicit argument Π is used. After combining these two equations

$$\dot{L} = \mathbf{J_1(\Pi)}\,\dot{X}_T$$
$$\dot{X}_T = \mathbf{J_2(\Pi)}\,\dot{\Pi}$$
$$\dot{L} = \mathbf{J_1(\Pi)\,J_2(\Pi)}\,\dot{\Pi} = \mathbf{J(\Pi)}\,\dot{\Pi}$$

Where the J (Π) is (6 x 6) Jacobian matrix.

3.2 Singularities in Selected Configuration: 6 SPS Parallel Manipulator in its Calculated Workspace [2]

Expression

$$\dot{L} = \mathbf{J_1(\Pi)\,J_2(\Pi)}\,\dot{\Pi} = \mathbf{J(\Pi)}\,\dot{\Pi}$$

Provides a transformation path from $\dot{\Pi}$ to $\dot{L}$

using the Jacobian matrix J (Π). It also shows that if J (Π) is not singular, then any variation of the velocity in link space can uniquely be transformed to the corresponding variation of the velocity in Cartesian space. Thus, the Jacobian acts as a bridge which allows a two-way transformation between the link space and the Cartesian space. Particularly if there is no movement in the link space i.e. $\dot{L}$ = 0. Then there is no movement in the Cartesian space and the parallel manipulator will be still. However, if at certain configurations J (Π) is singular and then the transformation path from the link space to the Cartesian space is blocked. In this case, even if there is no movement in the link space ($\dot{L}$= 0), the upper platform still possibly moves along some directions. In other words, at singularities, the parallel manipulator may gain extra degrees of freedom. The problem becomes even worse in that no force or torque in the Cartesian space can be transformed into the link space i.e. the Stewart platform is out of control at singular positions. It was suggested that taking manipulator redundancy may solve the problem. Actually redundancy can only move the original singularities from some points to the some other points. It cannot remove or even reduce the singular positions so problems originating due to singularities still remains in the system. Therefore, from the application viewpoint, investigating the conditions under which there will be singularities becomes more important and more useful. We need to check the singularity of the whole Jacobian matrix J in usable workspace. If value of det (J) is not zero in usable workspace then it can be said that there is no singularity in system.

3.3 Determination of Det (J) in Usable Workspace

J is square matrix of 6x6. Determinant of J can be easily find out for a pose of the platform. There are infinite numbers of poses of parallel manipulator in usable workspace. This is very necessary to check the value of Det J for whole usable workspace. For simplification of this process, a MATLAB loop code has been written to evaluate the value of det J for whole usable workspace. If the value of det J is zero at any pose of the platform. That pose is called as singular pose. Figure 5 shows the variation of det (J) value over the translational usable workspace of 100 mm x 100 mm as seen in x-y plane. The value of det (J) is lied between 2.9e+ 05 to 3.0598e+05 in usable workspace. So it can be concluded that there is no singularity in translational usable workspace. Similarly the value of det (J) has been calculated for rotational workspace as seen in Figure 6. Here also the value of det (J) is lied between 2.9e+ 05 to 3.0598e+05 in usable workspace. So it can be concluded that there is no singularity in rotational workspace also. Similarly the other value of Det (J) has been calculated for different usable workspace. None of sets of usable workspace has shown any type of singular position in the current manipulator system.

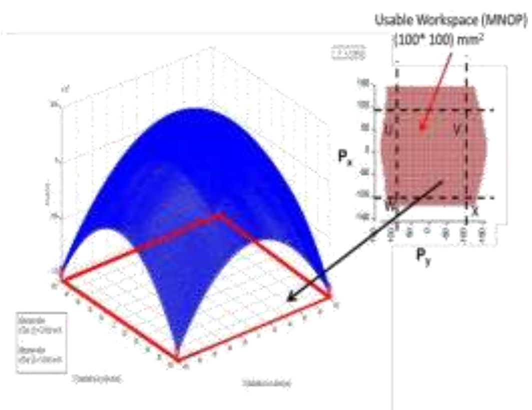

Fig. 5 Det (J) in Translational Usableworkspace as seen in x-y plane (Enclosed areas)

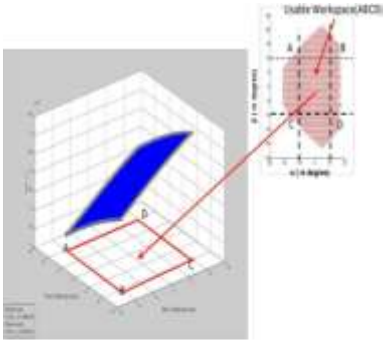

Fig. 6 Det (J) In Rotational Usable Workspace As Seen In X-Y Plane (Enclosed Areas)

IV. CONCLUSION

Nominal mathematical model of parallel manipulator has been discussed in this paper. Here in this paper our strategy was based on matrix analysis method to check and avoid the singularities in the usable workspace and analyze in advance and subsequently steering clear of them. Jacobian matrix has been formulated to check whether the singularity of parallel manipulator occurs in some certain postures or not. It could be determined by calculating the determinant of J. When the determinant J is equal to zero, singularities configuration will occur. A MATLAB loop code has been written to evaluate the value of det J for whole usable workspace and it has been found out that there is no any singular pose in current 6-DoF SPS parallel manipulator.

V. ACKNOWLEDGEMENT

This project is supported by the Centre for Design and Manufacturing, BARC and Homi Bhabha National Institute, BARC, Mumbai (India).

REFERENCES

[1] Agrawal S. K., “Workspace Boundaries ofIn-parallel manipulator Systems,” International Journal of Robotics and Automation, Vol 7, No 2, 1992, pp 94-99

[2] Gupta R.K. and Mishra V.K.,“Kinematicsand Workspace analysis of a 6-SPS Parallel Manipulator”,2014

[3] Tsai L W., “Robot Analysis: The Mechanicsof Serial and Parallel Manipulators”, NewYork: John Wiley & Sons, 1999, 223–257

[4] Bandyopadhyay S, Ashitava G.,“Analysisof configuration space singularities of closed-loop mechanisms and parallel manipulators”

[5] Sefrioui J, Gosselin C M.,“On thequadratic nature of the singularity curves of planar three degrees of freedom parallel manipulators”, Mech Mach Theory, 1995, 30(4):533–551

[6] Huang Z, Zhao Y S, Zhao T S., “AdvancedSpatial Mechanism”, Beijing: Higher EducationPress, 2006. 238–262

[7] Merlet J.P., "Parallel Robots", 2nd edition,Springer Publications, 2006.

[8] Lin and Ehmann, “ErrorModeling andCompensation of Stewart Platform”, Annals of CIRP, vol.7/3, pp.395-406, 1993.

[9] Veischegger, W. K., and Wu C., “Robot Accuracy Analysis”, IEEE proceedings ofinternational conference on cybernetics & Society, pp 425-430, 1985.