APPLICATION OF DISCRETE FOURIER TRANSFORM ON MULTISPECTRAL IMAGE OF BAREILLY REGION

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ABSTRACT

In this paper the multispectral image of Bareilly region is analyzed in Fourier domain using MatLab tool. The Fourier Transform is an important image processing algorithm which is used to decompose an image into its cosine and sine components. The output of the transformation represents the image in the frequency domain, while the input image is the spatial domain equivalent. In the Fourier domain image, each point represents a particular frequency contained in the spatial domain image. The DFT is the sampled Fourier Transform and therefore does not contain all frequencies forming an image, but only a set of samples which is large enough to fully describe the spatial domain image. The image in the spatial and Fourier domain is of the same size. The Fourier Transform is used when to access the geometric characteristics of a spatial domain image. The image in the Fourier domain is decomposed into its sinusoidal components, it is easy to observe or process certain frequencies of the image, thus influencing the geometric structure in the spatial domain. In image processing, only the magnitude of the Fourier Transform is displayed, as it contains most of the information of the geometric structure of the spatial domain image.

Keywords: Fourier Transform, DFT, Multispectral Image

I. INTRODUCTION

The Discrete Fourier Transform (DFT) is a specific kind of discrete transform, used in Fourier analysis. It transforms time domain function into another function, which is called the frequency domain representation, or simply the DFT, of the original function. The DFT requires an input function that is discrete. Such inputs are often created by sampling a continuous function. The discrete input function must also have a limited (finite) duration, such as one period of a periodic sequence or a windowed segment of a longer sequence.

The DFT of the sequence is a new periodic sequence and is related to the original sequence via a DFT inversion transform similar to the Inverse Fourier Transform. The DFT is widely employed in signal processing and related fields to analyze the frequencies contained in a sampled signal, to solve partial differential equations, and to perform other operations such as convolutions or multiplying large integers. The DFT turns out to be very usefulin spectral analysis of both Fourier series and Fourier transforms. A key enabling factor for these applications is the fact that the DFT can be computed efficiently in practice using a fast Fourier transform (FFT) algorithm.

FFT algorithms are so commonly employed to compute DFTs that the term "FFT" is often used to mean "DFT" in colloquial settings. DFT refers to a mathematical transformation or function, regardless of how it is computed, whereas "FFT" refers to a specific family of algorithms for computing DFTs.

Unfortunately, there are many different definitions of the DFT just as there are for the Fourier Transform. The sequence of N complex numbers is transformed into another sequence of N complex numbers according to the DFT formula:

$$F_n = \sum_{k=0}^{N-1} f_k \, e^{-2\pi i n k/N}, \text{ where } i = \sqrt{-1}.$$

But in Matlab we cannot use a zero or negative indices, so the sequences are $\{f_k\}_{k=1}^{N}$ and the DFT is computed as,

$$F_n = \sum_{k=1}^{N} f_k \, e^{-2\pi i (n-1)(k-1)/N}, \; n = 1, 2, \cdots, N.$$

For this case we have the formulae for the inverse Discrete Fourier Transform (IDFT) which gives

$$f_k = \sum_{n=1}^{N} F_n \, e^{2\pi i (n-1)(k-1)/N}, \; k = 1, 2, \cdots, N.$$

II. MATHEMATICAL DESCRIPTION OF DISCRETE FOURIER TRANSFORM OF IMAGE

The Fourier Transform is used in a wide range of applications, such as image analysis, image filtering, image reconstruction and image compression. The DFT is the sampled Fourier Transform and therefore does not contain all frequencies forming an image, but only a set of samples which is large enough to fully describe the spatial domain image. The image in the spatial and fourier domain are of the same size i.e. the number of frequencies corresponds to the number of pixels in the spatial domain region.

For a square image of size N×N, the two-dimensional DFT is given by:

$$F(k,l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i,j) \; e^{-\iota 2\pi \left(\frac{ki}{N} + \frac{lj}{N}\right)}$$

where f(a,b) is the image in the spatial domain and the exponential term is the basis function corresponding to each point F(k,l) in the Fourier space. The equation can be evaluated as: the value of each point F(k,l) is obtained by multiplying the spatial image with the corresponding base function and summing the result.

The basis functions are sine and cosine waves with increasing frequencies, i.e. F(0,0) represents the dc component of the image which corresponds to the average brightness and F(N-1,N-1) represents the highest frequency. In a similar way, the Fourier image can be re-transformed into the spatial domain. The inverse Fourier transform is given by:

$$f(a,b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k,l) \; e^{\iota 2\pi \left(\frac{ka}{N} + \frac{lb}{N}\right)}$$

To obtain the result for the above equations, a double sum has to be calculated for each image point. However, because the Fourier Transform is separable, it can be written as

$$F(k,l) = \frac{1}{N}\sum_{b=0}^{N-1} P(k,b)\, e^{-\iota 2\pi \frac{lb}{N}}$$

$$P(k,b) = \frac{1}{N}\sum_{a=0}^{N-1} f(a,b)\, e^{-\iota 2\pi \frac{ka}{N}}$$

By using these two formulas, the spatial domain image is first transformed into an intermediate image using N one-dimensional Fourier Transforms. This intermediate image is then transformed into the final image, again using N one-dimensional Fourier Transforms. Hence, the two-dimensional Fourier Transform can be expressed in terms of a series of 2N one-dimensional transforms which decreases the number of required computations.

Even with these computational savings, the ordinary one-dimensional DFT has N^2 complexity. This can be reduced to Nlog2N if we employ the *Fast Fourier Transform* (FFT) to compute the one-dimensional DFTs.

III. METHODOLOGY OR STEPS TO FIND DFT COEFFICIENTS OF IMAGE

In this paper we have transformed the multispectral image of Bareilly Region into its Fourier Domain using MatLab software. The Fourier Transform produces a complex number valued output image which can be displayed with two images, either with the real and imaginary part or with magnitude and phase.

- Read the image-First the image is read by using 'imread' function inMatLab
- Conversion of Colour image to Greyscale image:-After reading the image ,the image is converted into its grayscale image by using function 'rgb2gray'.
- Calculate the 2D DFT coefficients of grayscale image: After that 2D DFT coefficients of grayscale is calculated and estimated.
- After that logarithmic magnitude of 2D DFT coefficient is evaluated without any shifting to the pixel position.
- In the next step, logarithmic magnitude of 2D DFT coefficients of shifted pixel position is evaluated.
- After that phase of DFT coefficient is evaluated without any shifting to the pixel position.
- And at last, phase of DFT coefficient of shifted pixel position is evaluated.

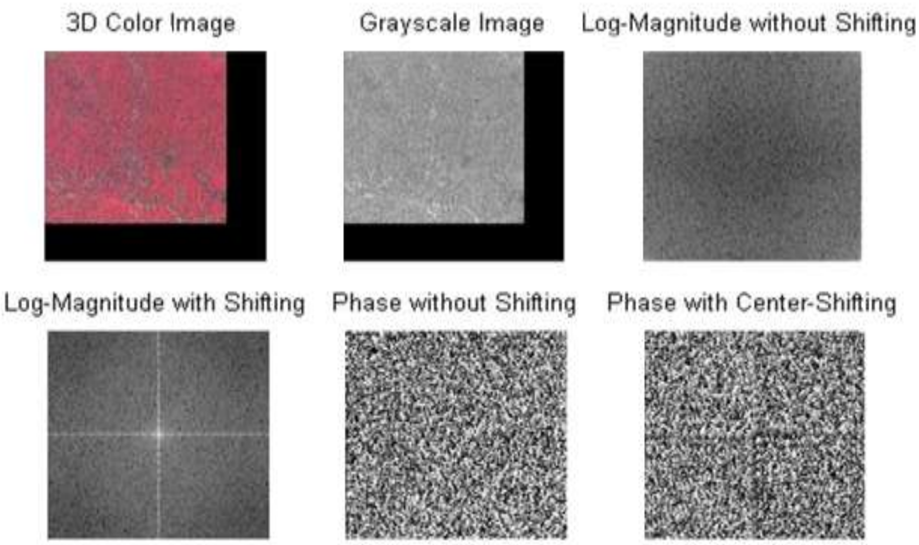

Figure1: Fourier Domain Analysis of Multispectral Image of Bareilly Region

The distance of the points to the center can be explained as follows:

The maximum frequency which can be represented in the spatial domain are two pixel wide stripe pairs (one white, one black).

$$f_{max} = \frac{1}{2 \text{ pixels}}$$

Hence, the two pixel wide stripes in the above image represent

$$f = \frac{1}{4 \text{ pixels}} = \frac{f_{max}}{2}$$

Thus, the points in the Fourier image are halfway between the center and the edge of the image, i.e. the represented frequency is half of the maximum.

3.1 Multispectral Images

A multispectral image consists of several bands of data. For visual display, each band of the image may be displayed one band at a time as a *grey scale image*, or in combination of three bands at a time as a colour composite image. Interpretation of a multispectral *colour composite image* will require the knowledge of the spectral reflectance signature of the targets in the scene. In this case, the spectral information content of the image is utilized in the interpretation.

A multispectral image is one that captures image data at specific frequencies across the electromagnetic spectrum. The wavelengths may be separated by filters or by the use of instruments that are sensitive to particular wavelengths, including light from frequencies beyond the visible light range, such as infrared. Spectral imaging can allow extraction of additional information the human eye fails to capture with its receptors for red, green and blue. It was originally developed for space-based imaging.

IV. CONCLUSION

The result of logarithmic magnitude of 2D DFT coefficients shows that the image contains components of all frequencies, but that their magnitude gets smaller for higher frequencies. Hence, low frequencies contain more image information than the higher ones. The transformed image in Fourier domain also tells us that there are two dominating directions in the Fourier image, one passing vertically and one horizontally through the center. These originate from the regular patterns in the background of the original image.

The value of each point determines the phase of the corresponding frequency. The phase of 2D DFT shows that as in the magnitude image, we can identify the vertical and horizontal lines corresponding to the patterns in the original image. The phase image does not yield much new information about the structure of the spatial domain image. Hence, in image processing, often only the magnitude of the Fourier Transform is displayed, as it contains most of the information of the geometric structure of the spatial domain image.

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