NEWTON-RAPHSON METHOD FOR SIMULATION OF MULTIPLE-EFFECT EVAPORATOR SYSTEM

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ABSTRACT

In the present Paper, A simplified model has been developed for the analysis of Triple-effect Multiple Effect Evaporator systems. For simplification, it is assumed that Boiling Point elevations are negligible. The model consists of linear equations which are solved using Newton-Raphson method.

Keywords: Simplified Model, Multiple Effect Evaporator, Newton-Raphson Method, Linear Equations

I. INTRODUCTION

Evaporation is an energy (heat)-driven separation process in which part of the solvent from a solution (with non-volatile solute) is removed by vaporization, resulting in a more concentrated solution. A solution containing a desired product is fed into the evaporator and it is heated by a heat source like steam. Because of the applied heat, the water in the solution is converted into vapor and is condensed while the concentrated solution is either removed or fed into a second evaporator for further concentration. The objective can be to concentrate the solution, to regenerate the solvent, or both. In many cases, however, the objective is to concentrate the solution. In these cases the solvent vapor is not regarded as product and it may or may not be recovered. Initially adopted in the production of salt with the aid of solar energy, evaporation has become the major Industrial concentration technique, water being the removed solvent in 99% of its applications. Evaporators are integral part of a number of process industries namely Pulp and Paper, Chlor-alkali, Sugar, Pharmaceuticals, Desalination, Dairy and Food processing, etc.

II. SINGLE EFFECT EVAPORATION

In a single-effect operation, as the name implies, only one evaporator is employed. The feed mixture to be separated or concentrated is introduced to the effect. The energy required to evaporate the solvent is supplied by the latent heat of vaporization given up by the steam upon condensing. The feed upon entering this effect must be heated to the boiling point temperature of the effect at the operating pressure. Then the solvent, generally water, is evaporated and removed as a vapor.

III. MULTIPLE EFFECT EVAPORATORS

The vapor driven off the liquid during evaporation contains considerable energy. In an evaporator with just a single effect this vapor is usually passed through a condenser and removed from the system. The energy efficiency of the process can be dramatically increased by reusing this vapor to evaporate the product further in
another effect. In order to do this the boiling temperature in the next effect must be sufficiently low to maintain an adequate temperature difference for the transfer of heat. This can be achieved by operating the second effect at a lower pressure. The reuse of the vapor in this way can be extended to a number of effects and is called multiple effect evaporation. For a multiple effect evaporator system the first effect is at the highest temperature and pressure and the last effect is at the lowest temperature and pressure.

In Multiple-Effect Evaporator, the vapor or steam produced in the first effect is introduced to the steam chest of the second effect and thus becomes the heating medium for the second effect. Similarly, the vapor from the second effect becomes the steam for the third effect.

Generally, the pure vapor above a solution is superheated because at a given pressure it condenses at a temperature below the boiling point temperature of the solution. The difference between the boiling point temperature of the solution and the condensation temperature of the vapor at the pressure of the vapor space is called the boiling point elevation of the effect.

To describe evaporator operation, the three terms, capacity, economy and consumption are commonly used. By capacity of an evaporator system is meant by the number of units of solvent evaporated per unit time. The economy of an evaporator system is the total number of units of solvent vaporized per unit of steam fed to the evaporator system. Steam consumption is the units of steam fed to the system per unit time, i.e. is the ratio of capacity to steam consumption.

**IV. MODELING OF MULTIPLE EFFECT EVAPORATOR SYSTEMS**

The design of evaporators is generally taken to mean the determination of the heat transfer area and the steam consumption required to effect a specified separation at a specified set of steady-state operating conditions.

**4.1 Assumptions**

1. Here we are describing the modeling equations for a triple effect evaporator system for the case where boiling point elevations are negligible.
2. The effect of composition on liquid enthalpy is neglected.
3. Vapor leaving an effect is at saturated condition.
4. Fouling resistance is negligible.
5. Each unit is perfectly mixed i.e., No pressure and velocity distribution and hence there is no need of momentum balance. The equations so obtained are generalized to include the case where boiling point elevations cannot be neglected.
6. Equal area effects are employed.

In the case of series operation with forward feed, depicted in figure 1, the thick liquor leaving the first effect becomes the feed for the second effect. For each effect added to the system, approximately one additional pound of solvent is evaporated per pound of steam fed to the first effect.
Figure: 1 A Triple-Effect Evaporator with Forward Feed. The Temperature Distribution Shown Is For A System with Negligible Boiling-Point Elevation

4.2 Nomenclature

F = feed rate, kg/hr;
L_j = Thick liquor rate, kg/hr, (where j= 1, 2, 3 for first, second and third effect respectively);
V_j = Vapor rate, kg/hr, (where j= 1, 2, 3 for first, second and third effect respectively);
X = Mass fraction of the solute in the feed;
x = Mass fraction of the solute in the thick liquor;
h_f = Enthalpy of the feed, kJ/kg;
h_j = Enthalpy of the thick liquor at the boiling point temperature of the evaporator, kJ/kg; for first, second and third effect respectively;
H_j = Enthalpy of the vapor at the boiling point of the evaporator, kJ/kg; for first, second and third effect respectively;
Q_j = Rate of heat transfer across the tubes (from the steam to the thick liquor), kJ/hr; for the first, second and third effect respectively.

4.3 Model Equations

Effect no. 1

A Total Material Balance is given by:-

\[ F = V_1 + L_1 \]  -----(1)

An enthalpy balance on the process stream yields

\[ F h_f + Q - V_1 H_1 - L_1 h_1 = 0 \]

Since \( V_1 = F - L_1 \), the previous equation can be rewritten as

\[ F ( h_f - h_1 ) + Q - ( F - L_1 ) ( H_1 - h_1 ) = 0 \]  -----(2)

Where \( H_j - h_j = \lambda_j \)

\( \lambda_j \) = The latent heat of vaporization of the solvent from the thick liquor at temperature \( T_1 \) and pressure \( P_1 \) (j = 1, 2, 3, the effect number)

For the first effect, by Enthalpy balance on the steam,

\[ Q_1 = V_0 ( H_0 - h_0 ) = \lambda_0 \]  -----(3)

And the rate of heat transfer is given by

\[ Q_1 = U_1 A ( T_0 - T_1 ) \]  -----(4)
By eliminating the value of \( Q \) by equation 3, equation 2 can be rewritten as
\[
F ( h_f - h_1 ) + Vo \lambda_o - ( F - L_1 ) \lambda_1 = 0
\]  ----(5)

The equation 4 and 5 may be stated in functional notation as follows

**Enthalpy Balance**
\[
f_1 = F ( h_f - h_1 ) + Vo \lambda_o - ( F - L_1 ) \lambda_1
\]  ----(6)

**Heat Transfer Rate**
\[
f_2 = U_1 A ( T_o - T_1 ) - Vo \lambda_0
\]  ----(7)

**Effect No. 2**

By Enthalpy Balance on steam,\( Q_2 = ( F - L_1 ) \lambda_1 \)

The Rate of heat transfer, \( Q_2 = U_2 A ( T_1 - T_2 ) \)

Hence, Enthalpy Balance
\[
f_3 = L_1 ( h_1 - h_2 ) + ( F - L_1 ) \lambda_1 - ( L_1 - L_2 ) \lambda_2
\]  ----(8)

**Heat Transfer rate**
\[
f_4 = U_2 A ( T_1 - T_2 ) - ( F - L_1 ) \lambda_1
\]  ----(9)

**Effect no. 3**

By Enthalpy Balance on steam, \( Q_3 = ( L_1 - L_2 ) \lambda_2 \)

The Rate of heat transfer, \( Q_3 = U_3 A ( T_2 - T_3 ) \)

Hence, Enthalpy Balance
\[
f_5 = L_2 ( h_2 - h_3 ) + ( L_1 - L_2 ) \lambda_2 - ( L_2 - L_3 ) \lambda_3
\]  ----(10)

**Heat Transfer rate**
\[
f_6 = U_3 A ( T_2 - T_3 ) - ( L_1 - L_2 ) \lambda_2
\]  ----(11)

**Component Material Balance**

The six independent equations can be solved for the six unknowns. In addition to these six independent equations, three additional equations that contain three additional independent variables \( x_1 \), \( x_2 \), and \( x_3 \) are given by
\[
FX - L_2 x_j = 0, \quad ( j = 1, 2, 3 )
\]  ----(12)

**V. USE OF THE NEWTON – RAPHSON METHOD FOR SOLVING EVAPORATOR DESIGN PROBLEMS**

The Newton – Raphson method consists of the repeated use of the linear terms of the Taylor series expansions of the functions \( f_1, f_2, f_3, f_4, f_5 \) and \( f_6 \) [ eq (6) – eq (11) ], i.e.,
\[
\frac{\partial f_1}{\partial V_o} (\Delta V_o) + \frac{\partial f_1}{\partial T_1} (\Delta T_1) + \frac{\partial f_1}{\partial L_1} (\Delta L_1) + \frac{\partial f_1}{\partial T_2} (\Delta T_2) + \frac{\partial f_1}{\partial L_2} (\Delta L_2) + \frac{\partial f_1}{\partial A} (\Delta A) = 0
\]  ----(13)

In equation 13, \( \Delta V_o = Y_{o,k+1} - Y_{o,k} \), \( \Delta T_1 = T_{1,k+1} - T_{1,k} \), \( \Delta T_2 = T_{2,k+1} - T_{2,k} \), \( \Delta L_1 = L_{1,k+1} - L_{1,k} \), \( \Delta L_2 = L_{2,k+1} - L_{2,k} \), \( \Delta A = A_{k+1} - A_k \)

Where subscripts \( k \) and \( k+1 \) denote the \( k \)th and \( k+1 \)th trials. These six equations may be stated in compact form by means of the following matrix equation
\[
J_k \Delta X_k = -f_k
\]  ----(14)

Where \( J_k \) is called the Jacobian Matrix and \( \Delta X_k = X_{k+1} - X_k = [ \Delta V_o, \Delta T_1, \Delta L_1, \Delta T_2, \Delta L_2, \Delta A ]^T $
The subscripts k and k+1 denote that elements of the matrices carrying these subscripts are those given by the kth and k+1st trials, respectively. The display of the elements of $J_k$ and $f_k$ is as follows:

$$J_k = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_1} \\
\frac{\partial f_1}{\partial x_2} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_2} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_1}{\partial x_n} & \frac{\partial f_2}{\partial x_n} & \cdots & \frac{\partial f_m}{\partial x_n}
\end{bmatrix}$$

$$f_k = \begin{bmatrix}
f_1 \\
f_2 \\
f_3 \\
f_4 \\
f_5 \\
f_6
\end{bmatrix}$$

$$J = \begin{bmatrix}
\lambda_1 & -C_p & 0 \\
0 & L_1C_p & b_{12} \\
0 & U_2A & \lambda_2 \\
0 & 0 & L_2C_p \\
0 & 0 & \lambda_3 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$$

Where, $b_{13} = C_p(T_1 - T_2) - (\lambda_1 + \lambda_2)$ & $b_{55} = C_p(T_2 - T_3) - (\lambda_2 + \lambda_3)$

**VI. SOLUTION OF A TEE PROBLEM BY NEWTON-RAPHSON METHOD**

We have to assume the value of following terms for the calculation procedure to design triple effect evaporator,

1- $(T_o - T_1)$, $(T_1 - T_2)$, and $(T_2 - T_3)$
2- Solvent evaporated in first effect
3- Solvent evaporated in second effect
4- Solvent evaporated in third effect
5- Area, A
6- $V_o$, Vapor flow rate of steam

The following scaling procedure should be used:

1- Each functional equation is divided by the product $F\lambda_o$, and the new functional expression so obtained are denoted by $g_j$ ($j = 1, 2, \ldots, 6$)

$$g_j = f_j / F \lambda_o$$

2- All flow rates are expressed as a function of the feed rate $F$, that is $L_j = l_j F$ and $V_j = v_j F$.

3- All the temperatures are expressed as a function of the steam temperature, $T_F = u_j T_o$, which defines the fractional temperature $u_j$.

4- The area of each effect is expressed as a fraction of a term proportional to the flow rate in the following manner:-
After this scaling procedure has been applied to the functional expressions, the matrices $J_k$, $\Delta X_k$, $ndf_k$ take the following terms:

\[
J_k = \begin{bmatrix}
0 & 0 & -\frac{\lambda_1}{\lambda_2} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_1}{\lambda_2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\lambda_1}{\lambda_2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\lambda_1}{\lambda_2} \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

$\Delta X_k \cong \begin{bmatrix}
\Delta v_0 \\
\Delta u_1 \\
\Delta l_1 \\
\Delta u_2 \\
\Delta l_2 \\
\Delta a \\
\end{bmatrix}$

$\mathbf{f}_k = \begin{bmatrix}
\mathbf{g}_1 \\
\mathbf{g}_2 \\
\mathbf{g}_3 \\
\mathbf{g}_4 \\
\mathbf{g}_5 \\
\mathbf{g}_6 \\
\end{bmatrix}$

Where the elements of $J_k$ consist of the partial derivatives of the $g_j$’s with respect to the new set of variables ($v_0$, $u_1$, $u_2$, $l_1$, $a$)

\[
b_{12} = -C_p T_o / \lambda_o, \quad b_{13} = -1; C_p T_o / \lambda_o, \quad b_{22} = -U_1 a T_o / 50 \lambda_o, \quad b_{23} = U_1 a T_o / 50 \lambda_o, \quad b_{32} = -U_1 a T_o / 50 \lambda_o, \quad b_{33} = U_1 a T_o / 50 \lambda_o,
\]

Convergence to within about six significant numbers is achieved in further trials. We get the value of $v_0$, $u_1$, $u_2$, $l_1$, $l_2$, $a$.

VII. CONCLUSION

On the basis of above procedure we can design multiple effect evaporator using Newton-Raphson method. For Triple-effect evaporator, we can directly put the variables into the final solution and can find the solution for a specific practical problem. The formulation of the solution of problems in terms of the Newton-Raphson method is helpful because it forces one to display the independent equations and the independent variables.

VIII. RECOMMENDATIONS

1. This procedure can be further extended to include more effects. Here forward mixing is considered.
2. Backward mixing can also be taken into consideration.
3. The model should further be simulated with operating strategies i.e, condensate flash, feed and product vapor bleeding etc. These strategies are beneficial for increasing the steam economy of a MEE system and they work as Energy reduction schemes.
REFERENCES


